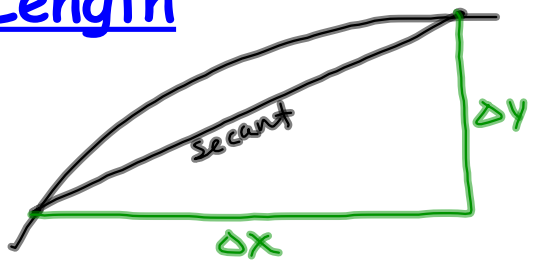
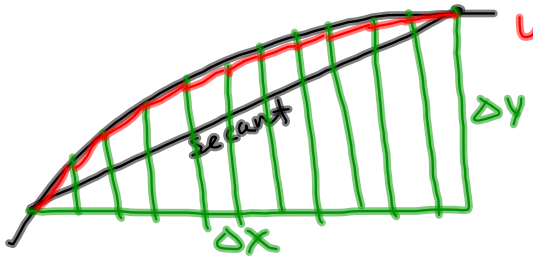
Arc Length



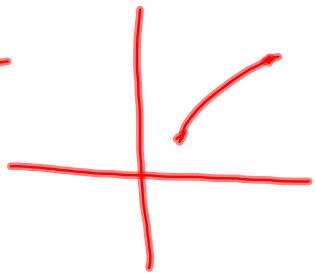
$$(\text{sec length})^2 = \Delta x^2 + \Delta y^2$$



whole bunch of secants $\approx$ length of curve

$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

ex1 $y = x^{3/2}$ $(1,1) \to (2, 2\sqrt{2})$



$f(x) = x^{3/2}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$

$$L = \int_1^2 \sqrt{1 + \left(\tfrac{3}{2}x^{\frac{1}{2}}\right)^2}\,dx = \frac{4}{9}\int_1^2 \sqrt{1 + \tfrac{9}{4}x}\,\tfrac{9}{4}\,dx$$

let $u = 1 + \frac{9}{4}x$

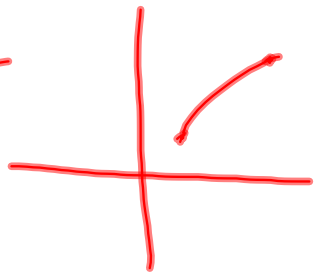
$du = \frac{9}{4}dx$

$$= \frac{4}{9}\cdot\frac{\left(1+\frac{9}{4}x\right)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_1^2 = \frac{8}{27}\left(1+\tfrac{9}{4}x\right)^{3/2}\Bigg|_1^2 = \frac{8}{27}\left(\left(\tfrac{11}{2}\right)^{3/2} - \left(\tfrac{13}{4}\right)^{3/2}\right)$$

≈ 2.09

ex1 $y = x^{3/2}$ $(1,1) \to (2, 2\sqrt{2})$

$x = y^{2/3}$ $x' = \frac{2}{3} y^{-\frac{1}{3}}$

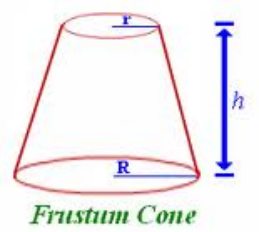


$$\int_1^{2\sqrt{2}} \sqrt{1 + \tfrac{4}{9} y^{-\frac{2}{3}}}\, dy = \text{something}$$

Area of a Surface of Revolution

please notice that it says "Area" no "Volume" and it says "surface" not "solid"...k?

see pretty pictures on page 485

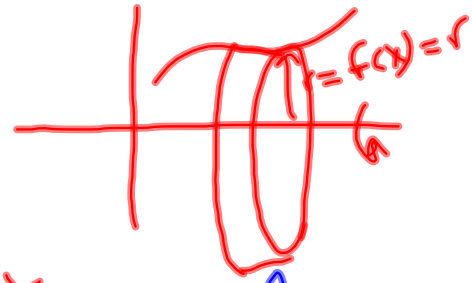


Frustum Cone

surface area is just a bunch of these "segments" of area added together. As the h approaches 0, the error goes away.

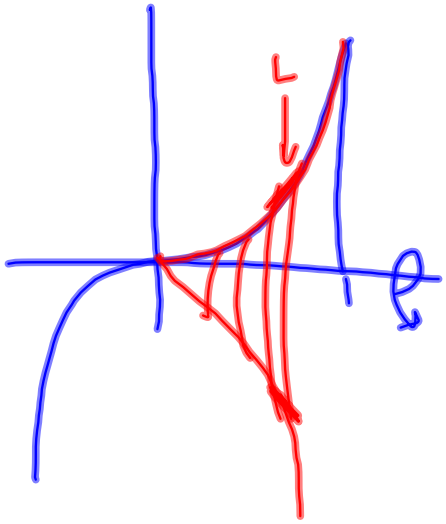
$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

$$SA = \int_a^b 2\pi f(x) \sqrt{1 + f'(x)^2}\, dx$$



Slant height

e1/ $y=x^3 \quad y'=3x^2$



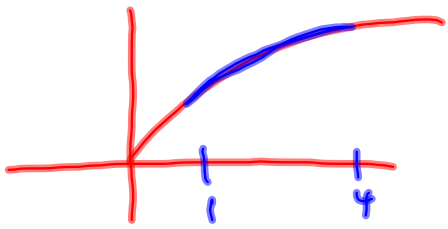
$$SA=\int_0^1 2\pi x^3\sqrt{1+(3x^2)^2}\,dx$$

$$\frac{2\pi}{36}\int_0^1 \sqrt{1+9x^4}\,x^3dx\cdot 36$$

$$\frac{\pi}{18}\frac{(1+9x^4)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_0^1=\frac{\pi}{27}\left(10\sqrt{10}-1\right)$$

#2 pg 488

$y = \sqrt{x} \quad 1 \le x \le 4$



$y' = \frac{1}{2\sqrt{x}}$

$(y')^2 = \frac{1}{4x}$

$$\int_1^4 2\pi\sqrt{x}\left(\sqrt{1+\tfrac{1}{4x}}\right)dx$$

$$2\pi\int_1^4 \sqrt{x+\tfrac{1}{4}}\,dx$$

$$2\pi\left.\frac{\left(x+\frac{1}{4}\right)^{3/2}}{3/2}\right|_1^4 = \frac{4}{3}\pi\left(\left(\frac{17}{4}\right)^{3/2}-\left(\frac{5}{4}\right)^{3/2}\right)$$

$$= \frac{\pi}{6}\left(17\sqrt{17}-5\sqrt{5}\right)$$

P. 484 #3,4
P. 488 #3,7