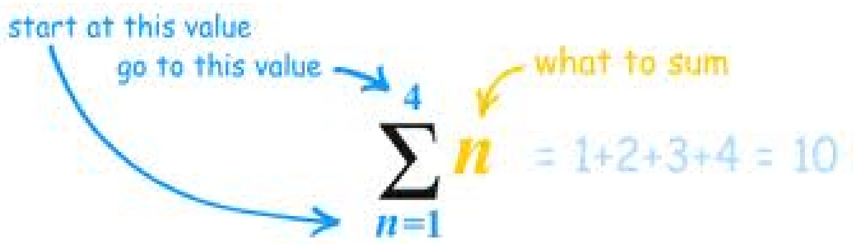
sigma notation Σ



$$\sum_{k=4}^{8} k^3 = 4^3 + 5^3 + 6^3 + 7^3 + 8^3$$

$$\sum_{K=1}^{5} 2K = 2+4+6+8+10$$

$$\sum_{K=1}^{5} 2K-1 = 1+3+5+7+9 = \sum_{k=0}^{4} 2k+1$$

$$\sum_{K=1}^{5} (-1)^K(2K-1) = -1+3-5+7-9$$

$$\sum_{k=1}^{3} k \sin \frac{k\pi}{5} = \sin \frac{\pi}{5} + 2 \sin \frac{2\pi}{5} + 3 \sin \frac{3\pi}{5}$$

$$\sum_{i=1}^{5} 2 = 2+2+2+2+2$$

$$\sum_{i=1}^{5} 2k = 2k+2k+2k+2k+2k$$

$$\sum_{j=0}^{2} x^3 = x^3+x^3+x^3$$

Jul 22-2:07 PM

$$\sum_{k=1}^{5} 2k = 2+4+6+8+10$$

$$= \sum_{k=0}^{4}(2k+2) = \sum_{k=2}^{6}(2k-2)$$

ex 2

$$\sum_{k=3}^{7} 5^{k-2}$$

lower limit = 0, not 3

so let $j = k-3$

$k = j+3$

$$\sum_{j=0}^{4} 5^{(j+3)-2} = \sum_{j=0}^{4} 5^{j+1} = \sum_{k=0}^{4} 5^{k+1}$$

Jul 22-2:09 PM

page 399: algebraic properties of sigma notation

$$\sum_{k=1}^{n} c\,a_n = c\sum_{k=1}^{n} a_n$$

$$\sum_{k=1}^{n} (a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$$

Jul 22-2:13 PM

theorems involving sigma notation

$$\sum_{k=1}^{n} k = 1+2+\ldots+n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1+4+9+\ldots+n^2 = \frac{(2n+1)(n+1)(n)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1+8+27+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

ex3,4

$$\sum_{k=1}^{30} k(k+1) = \sum_{k=1}^{30} k^2+k = \sum_{k=1}^{30} k^2 + \sum_{k=1}^{30} k$$

$$= \frac{(n)(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{(30)(31)(61)}{6} + \frac{30(31)}{2}$$

$$= 9920$$

$$\sum_{k=1}^{n}(3+k)^2 = \sum_{k=1}^{n}(9+6k+k^2)$$

$$= \sum_{k=1}^{n}9 + \sum_{k=1}^{n}6k + \sum_{k=1}^{n}k^2$$

"Closed sum"

$$= 9n + 6\sum_{k=1}^{n}k + \frac{n(n+1)(2n+1)}{6}$$

$$= 9n + 6\left(\frac{n(n+1)}{2}\right) + \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}n^3 + \frac{7}{2}n^2 + \frac{73}{6}n$$

(after some simplifying) ☺

44) $\sum_{k=1}^{50}\left(\frac{1}{k}-\frac{1}{k+1}\right)$

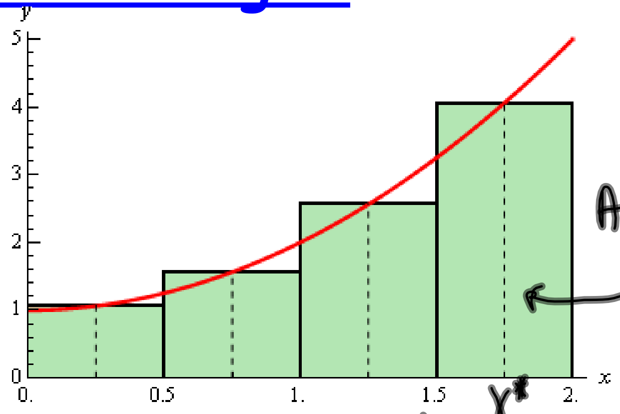
$$=\left[\frac{1}{1}-\frac{1}{2}\right]+\left[\frac{1}{2}-\frac{1}{3}\right]+\left[\frac{1}{3}-\frac{1}{4}\right]$$

$$=\left[1-\cancel{\frac{1}{2}}\right]+\dots+\left[\cancel{\frac{1}{50}}-\frac{1}{51}\right]$$

"telescoping sum"

$$=1-\frac{1}{51}=\frac{50}{51}$$

The definite integral



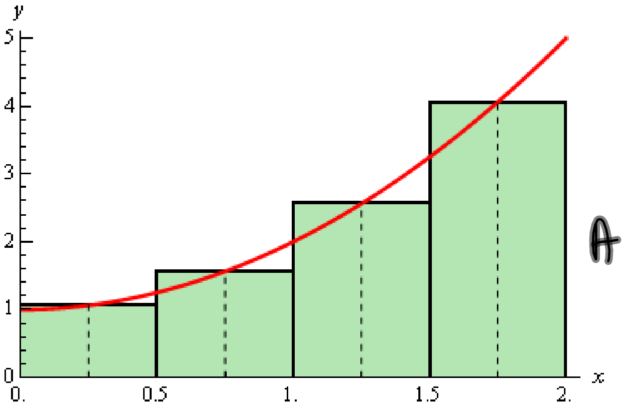
$A = f(x^*)\,h$

$x \quad x^*$

as $n \to \infty \quad h \to 0$

$x^* \to x$

regardless of the choice of x* for the height of the rectangle (left,center,rigth) all these x*'s approach x as the width of the interval approaches 0



$A = f(x^*)\,h$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

look at the sum of the areas of the rectangles. As the number of rectangles gets large without bound, the limit of that sum is the area under the curve

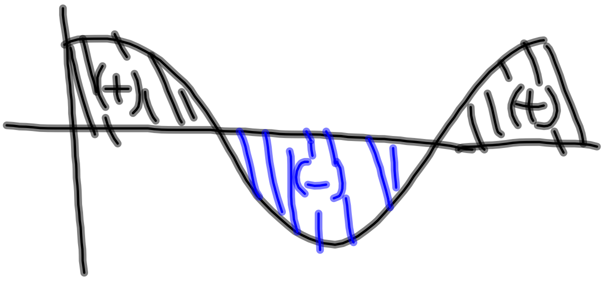
A = A

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$$

Riemann sum
"definite" integral

$[a, b]$

look at fig 7.5.8 top of page 408



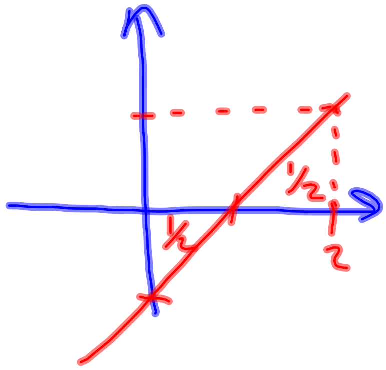
in a sum those will cancel!

Jul 22-2:02 PM

ex3 $y = x - 1$

$$\int_0^2 (x-1)\,dx = 0$$

$$\int_0^1 (x-1)\,dx = -\frac{1}{2}$$



7.5.3 pg 411

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

7.5.4 pg 411

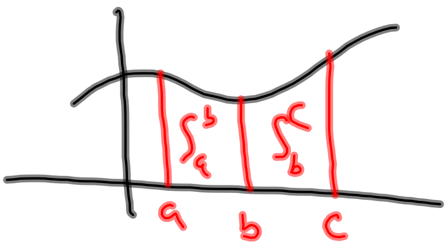
$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b \left(f(x) \pm g(x)\right)dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

Jul 23-10:50 AM

7.5.5 p412

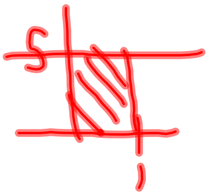
$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx \qquad a<b<c$$



ex 5 p 413

$$\int_0^1 5 - 3\sqrt{1-x^2}\,dx$$

$$= \int_0^1 5\,dx - 3\int_0^1 \sqrt{1-x^2}\,dx$$



$\frac{1}{4}\pi(1)^2$

$$= 5 - 3\left(\frac{\pi}{4}\right) = 5 - \frac{3\pi}{4}$$

$y = \sqrt{1-x^2}$
$y^2 = 1-x^2$
$x^2+y^2=1$

Jul 23-10:53 AM

HW:

p402 1b,5,19,25,43

p414 17c,19c,21,23,25,33,37

Jul 23-10:56 AM