Inverse functions

functions are inverses of each other if the "undo" what the other does. Sort of like 3-yr-olds.

$f(x) = 3x + 1$

$f(2) = 3(2) + 1$

$= 7$

$g(x) = \frac{(x-1)}{3}$

$g(7) = \frac{7-1}{3}$

$= 2$

$f(x)=3x+1$

$g(x)=\frac{(x-1)}{3}$

$(f\circ g)(x)=f\left(\frac{x-1}{3}\right)=3\left(\frac{x-1}{3}\right)+1=x-1+1=x$

$(g\circ f)(x)=g(3x+1)=\frac{(3x+1)-1}{3}=\frac{3x}{3}=x$

$f(x) = 3x+1 \Rightarrow g^{-1}(x)$

$g(x) = \frac{(x-1)}{3} \Rightarrow f^{-1}(x)$

$$f^{-1}(x) \neq \frac{1}{f(x)}$$

$$[f(x)]^{-1} = \frac{1}{f(x)}$$

$f(x) = x^2$
$g(x) = \sqrt{x}$

is $g(x) = f^{-1}(x)$?

$f(x)=x^2$

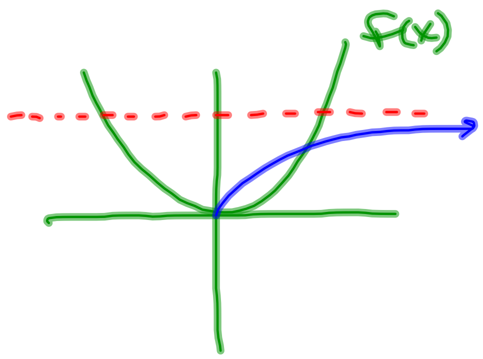
$g(x)=\sqrt{x}$ is $g(x)=f^{-1}(x)$?

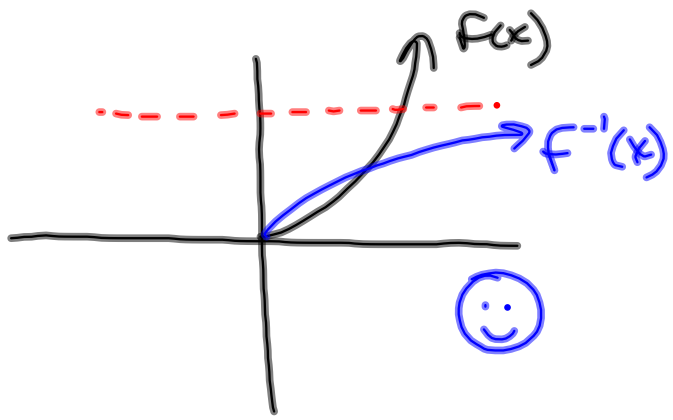
$(f\circ g)(x)=f(\sqrt{x})=(\sqrt{x})^2=x$

$(g\circ f)(x)=g(x^2)=\sqrt{x^2}=|x|$ NO

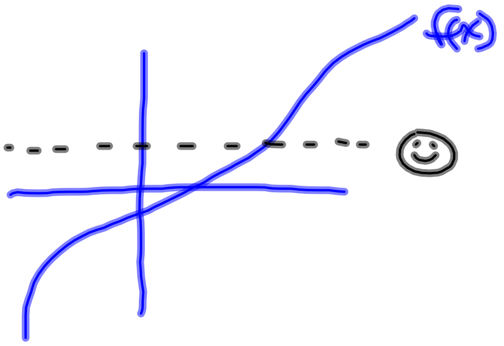
$f(x)=x^2 \quad x: x\geq 0$

$f^{-1}(x)=\sqrt{x}$



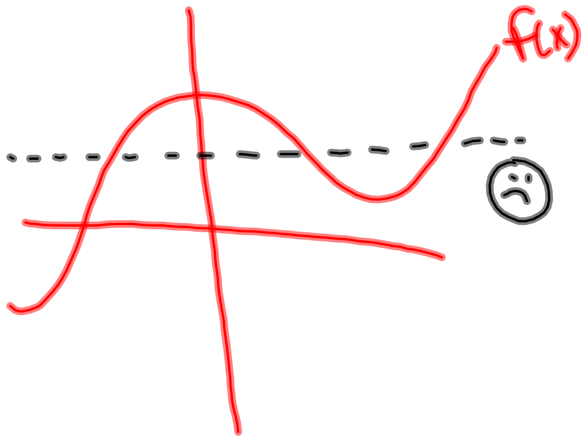


to have an inverse, a function must pass the HORIZONTAL line test... it must be a 1-1 function.



all tangents have (+) slopes

$\therefore f'(x) > 0$



some (+) slope tans + some (−) slope tans

no sign restriction on $f'(x)$

if $f'(x) > 0$ $\left(\text{or } f'(x) < 0\right)$
on the entire Domain of $f(x)$
then $f^{-1}(x)$ exists

$f(x) = 3x+1$

$y = 3x+1$

$x = 3y+1$

$x-1 = 3y$

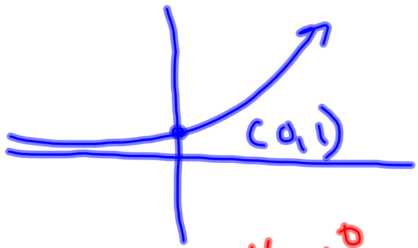
$\frac{x-1}{3} = y$

$f^{-1}(x) = \frac{x-1}{3}$

a function and its inverse are reflections of each other in the line y=x the exponential (e^x) and logarithmic ($\ln x$) functions.

$y = a^x$

$$\begin{cases} y = 2^x \\ y = \pi^x \\ y = e^x \end{cases}$$



$y = a^0$

$y = 1$

$e = 2.718281828459045\ldots$

$y = \log_a b \implies a^y = b$

equivalent equations

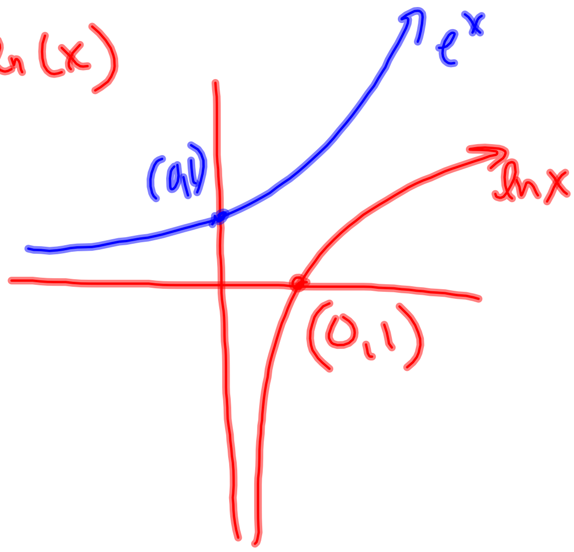
$y = \log_a x$ and $y = a^x$ are inverses

$a^y = x$

flip x, y to get inverse

$a^x = y$

the natural logarithm $\ln(x)$
is just $\log_e x$



$\ln(0) = \log_e(0) =$ u.d.
$\ln(1) = \log_e(1) = 0$
$\ln e = \log_e e = 1$
$\ln \frac{1}{e} = \log_e \frac{1}{e} = -1$
$\ln e^2 = \log_e e^2 = 2$ $\quad \left(\ln e^2 = 2 \log e = 2 \cdot 1 = 2 \right)$

Properties (algebraic) of logs

$\log_c(ab) = \log_c a + \log_c b$

$\log_c\left(\frac{a}{b}\right) = \log_c a - \log_c b$

$\log_c(a^b) = b\log_c a$

$(\log_c a)^2 = (\log_c a)(\log_c a)$

$\log_c\left(\frac{1}{a}\right) = -\log_c a$

$\log_r(a \pm b)$ no form

ex 1 $\ln(x+1)=5 \Rightarrow x = 147.41$

$e^5 = x+1$

$x = e^5 - 1 = 147.41$

ex 2 $P = 75e^{-t/125}$

$e^{-x} = \frac{1}{e^x}$ ↓

$7 = 75e^{-t/125}$

$\ln\left(\frac{7}{75} = e^{-t/125}\right)$

$\ln\frac{7}{75} = -\frac{t}{125}$ → $-t = (125)\ln\frac{7}{75}$

$= 296$ days

ex 3

$$\frac{e^x - e^{-x}}{2} = 1$$

$$e^x - e^{-x} = 2$$

$$e^x e^x - 1 = 2e^x$$

$$e^{2x} - 1 - 2e^x = 0$$

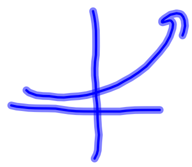
$$e^{2x} - 2e^x - 1 = 0$$

$$(e^x)^2 - 2(e^x) - 1 = 0$$

$$e^x = \frac{2 \pm \sqrt{4-4(-1)}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$e^x = 1+\sqrt{2}$$



$$x = \ln\left(1+\sqrt{2}\right)$$

$$x = .881$$

HW:

Pg 233 1c,11,17
Pg 243 5,9,13,25,27,34b(don't cheat!)