Implicit differentiation

$$xy = 1$$

$$y = \frac{1}{x} \quad y' = -\frac{1}{x^2}$$

$$\frac{d\,xy}{dx} = \frac{d\,1}{dx}$$

$$x\frac{dy}{dx} + y\frac{dx}{dx} = 0$$

$$x\frac{dy}{dx} + y = 0$$

$$xy' = -y$$

$$y' = -\frac{y}{x} \quad \checkmark$$

$$y' = -\frac{\frac{1}{x}}{x}$$

$$y' = -\frac{\frac{1}{x}}{x}$$

$$y' = -\frac{1}{x}\frac{1}{x} = -\frac{1}{x^2} \quad ☺$$

Ex2

$$5y^2 + \sin y = x^2$$

$$\frac{d(5y^2 + \sin y)}{dx} = \frac{d\,x^2}{dx}$$

$$5\frac{d\,y^2}{dx} + \frac{d\sin y}{dx} = 2x$$

$$5\cdot 2y\cdot\frac{dy}{dx}^{*} + (\cos y)\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(10y + \cos y) = 2x$$

$$^{*}\ \frac{d\,y^2}{dx} = 2y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x}{10y + \cos y}$$

ta dá!!!

ex3 $y^2-x+1=0$

$$\frac{d\, y^2-x+1}{dx} = \frac{d\,0}{dx}$$

$$\frac{dy^2}{dx}-\frac{dx}{dx}+\frac{d1}{dx}=0$$

$$2y\cdot\frac{dy}{dx}-1=0$$

$$\frac{dy}{dx}=\frac{1}{2y}$$

$(2,-1)$

$\frac{dy}{dx}=-\frac{1}{2}$ ✓

$(2,1)$

$\frac{dy}{dx}=\frac{1}{2}$ ✓

derivatives of logarithmic and exponential function.

$$\frac{d(\ln x)}{dx} = \frac{1}{x} \qquad x > 0$$

$$\frac{d \ln u}{dx} = \frac{1}{u}\frac{du}{dx}$$

$$f(x) = \ln(3x^2)$$

$$f'(x) = \frac{1}{3x^2} \cdot 6x = \frac{2}{x}$$

ex 6

$$y = \frac{x^2 \sqrt[3]{7x-14}}{(1+x^2)^4}$$

$$\ln y = \ln\left(\frac{x^2 \sqrt[3]{7x-14}}{(1+x^2)^4}\right)$$

$$\ln y = \ln\left(x^2 \sqrt[3]{7x-14}\right) - \ln(1+x^2)^4$$

$$= \ln x^2 + \ln(7x-14)^{1/3} - 4\ln(1+x^2)$$

$$\ln y = 2\ln x + \frac{1}{3}\ln(7x-14) - 4\ln(1+x^2)$$

$$\ln y = 2\ln x + \frac{1}{3}\ln(7x-14) - 4\ln(1+x^2)$$

$$\frac{1}{y}\frac{dy}{dx} = 2\left(\frac{1}{x}\right) + \frac{1}{3}\frac{1}{7x-14}(7) - 4\frac{1}{1+x^2}2x$$

$$\frac{dy}{dx} = \left(\frac{2}{x} + \frac{7}{3}\frac{1}{7x-14} - \frac{8x}{1+x^2}\right)y$$

plug in y

exponentials

$$y = b^x \Longrightarrow \log_b y = x$$

page 256 eg-3

$$\frac{d \log_b y}{dx} = \frac{d\,x}{dx}$$

$$\frac{1}{y \ln b} \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = y \ln b$$

$y = 7^x$

$\frac{dy}{dx} = 7^x \ln 7$

$y = e^x$

$\frac{dy}{dx} = e^x \ln e$

$= e^x$

hey!!??

$\ln x = \log_e x$

$\ln e = \log_e e$

$y = \log_e e$

$e^y = e$

$y = b^u$

$$\frac{dy}{dx} = b^u \ln b \, \frac{du}{dx}$$

$y = \pi^{3x^2}$

$$\frac{dy}{dx} = \pi^{3x^2} (\ln \pi) \cdot 6x$$

$y = e^{(2x-1)}$

$\frac{dy}{dx} = e^{2x-1}(\ln e) \cdot 2$

$= 2e^{2x-1}$

Ex. 7 pg 259

$y = 2^{\sin x}$ $\quad y' = 2^{\sin x} \ln 2 \cos x$

$y = e^{-2x}$ $\quad y' = e^{-2x}(-2)$

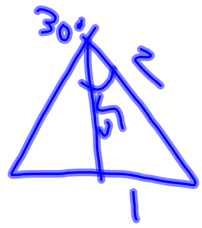
$y = e^{x^3}$ $\quad y' = e^{x^3}(3x^2)$

$y = e^{\cos x}$ $\quad y' = e^{\cos x}(-\sin x)$

Inverse Trig Functions (pg 262)

Just look at def^{n}s + graphs.

$\csc^{-1}(2) = \pi/6$

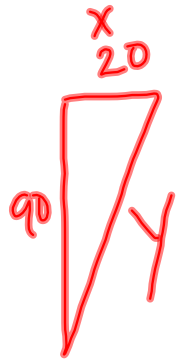


Derivatives of Inverse Trig Functions
(pg 266)

ex 3... questions?

Related Rates!

ex 2 p 271, ex 4 p 273



<u>ex2</u>

$t =$ # secs from 2nd base

$x =$ dist to 3rd

$y =$ dist to Home

find $\frac{dy}{dt}$ $\quad x_0 = 20$

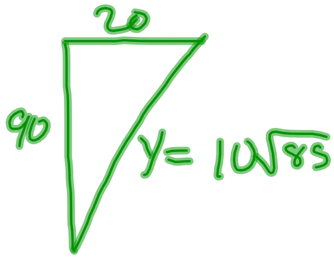
$\frac{dx}{dt} = -30$

$$y^2 = x^2 + 90^2$$

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt} + 0$$

$$\frac{dy}{dt} = \frac{\cancel{2}x\frac{dx}{dt}}{\cancel{2}y}$$

$$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$$



$x = 20$

$\frac{dx}{dt} = -30$

$$\frac{dy}{dt} = \frac{20}{10\sqrt{85}}(-30)$$

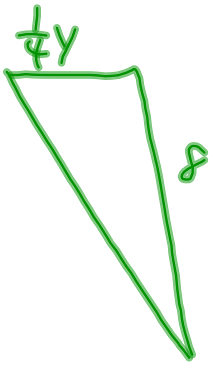
$$= -\frac{60}{\sqrt{85}} = -6.51 \text{ ft/sec}$$

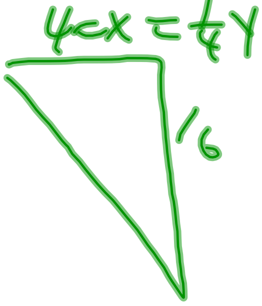
<u>ex4</u> t, V, d, x $\frac{dV}{dt} = 2$ $y = 8$

$V = \frac{1}{3}\pi x^2 y$ $\frac{dy}{dt} = ?$

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(x^2\frac{dy}{dt} + y\,2x\frac{dx}{dt}\right)$$

no info on $\frac{dx}{dt}$ so.... get x out first





$$V = \frac{1}{3}\pi\left(\frac{1}{4}y\right)^2 y$$
$$= \frac{\pi}{48}y^3$$
$$\frac{dv}{dt} = \frac{\pi}{48}\cdot 3y^2\frac{dy}{dt}$$
$$\frac{dy}{dt} = \frac{dv}{dt}\left(\frac{48}{3y^2\pi}\right)$$
$$= \frac{16}{y^2\pi}\frac{dv}{dt}$$

$$\left.\frac{dy}{dt}\right|_{y=8} = \frac{16}{64\pi}(-2)$$
$$= -\frac{1}{2\pi} \text{ cm/min}$$

L'Hopital's rule (indeterminate forms for limits)

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{or} \quad \frac{\infty}{\infty}$$

$$\lim_{x \to a} \frac{f'(x)}{g'(x)} = \lim_{x \to a} \frac{f(x)}{g(x)} \quad \text{iff}$$

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{0}{0}$$ ☹

$f'(x) = 2x$

$g'(x) = 1$

$$\lim_{x \to 2} \frac{2x}{1} = 4$$

$$\lim_{x \to 0^-} \frac{\tan x}{x^2} = \frac{0}{0}$$ ☹

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\lim_{x \to 0^-} \frac{\sec^2 x}{2x} = \frac{1}{0^-} = \infty$$

HW:

Pg 253 9,11,21,27
Pg 260 1,13,21,31,35
Pg 267 1a,2a,5,7d,9a,11,21,29
Pg 275 21,29
pg 284 3,9,19,21,27