Integration by Parts

"Parts is parts!"

$$\frac{d\left(f(x)\cdot g(x)\right)}{dx} = f(x)g'(x) + g(x)f'(x)$$

$$\int \frac{d\left(f(x)\cdot g(x)\right)}{dx}\,dx = \int f(x)g'(x)\,dx + \int g(x)f'(x)\,dx$$

$$f(x)\cdot g(x) = \int f(x)g'(x)\,dx + \int g(x)f'(x)\,dx$$

$$f(x) \cdot g(x) = \int f(x) g'(x)\,dx + \int g(x) f'(x)\,dx$$

$$\int f(x) g'(x)\,dx = f(x) \cdot g(x) - \int g(x) f'(x)\,dx$$

let $u = f(x)$
$v = g(x)$

$$\int u\,dv = u \cdot v - \int v\,du$$

ex1

$$\int x e^x dx$$

let $u = x$ $\Rightarrow$ $du = dx$

$dv = e^x dx$ $\Rightarrow$ $v = e^x$

$$\int u\,dv = xe^x - \int e^x dx = xe^x - e^x + C$$

$y = xe^x - e^x + C$

$y' = xe^x + e^x \cdot 1 - e^x$

$y' = xe^x \cancel{+e^x} \cancel{-e^x}$

let $u = e^x$ $\quad du = e^x dx$

$dv = x\,dx$ $\quad v = \frac{x^2}{2}$

$$\int u\,dv = \frac{x^2}{2}e^x - \int \frac{x^2}{2} e^x dx$$

<u>ex2</u> $\int x^2 e^{-x}\,dx$

let $u = x^2$, $dv = e^{-x}dx$ $\Big\} \Rightarrow$ $du = 2x\,dx$, $v = -e^{-x}$

$$\int x^2 e^{-x}\,dx$$

$$\int u\,dv = -x^2 e^{-x} + \int e^{-x}(2x\,dx)$$

$$2\int e^{-x} x\,dx$$

$u = x$, $dv = e^{-x}dx$ $\quad$ $du = dx$, $v = -e^{-x}$

$$2\int u\,dv = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + C \times 2$$

$$-x^2e^{-x}-2xe^{-x}-2e^{-x}+C = \int x^2e^{-x}dx$$

applying integration by parts more than once in the same problem is a common occurrence, so don't blow it off :)

3) $\int \ln x \, dx$

let $u = \ln x$ $\quad du = \frac{1}{x} dx$

$dv = dx$ $\quad v = x$

$$\int u\,dv = x\ln x - \int x \frac{1}{x} dx$$

$$= x\ln x - x + C$$

$y = x\ln x - x + c$

$y' = x\frac{1}{x} + \ln x \cdot 1 - 1$

$y' = 1 + \ln x - 1$

$y' = \ln x$

4) $\int e^x \cos x\,dx$

let $u = e^x$ $\quad du = e^x dx$

$dv = \cos x\,dx$ $\quad v = \sin x$

$\int u\,dv = \sin x\, e^x - \underline{\int e^x \sin x\,dx}$

let $u = e^x$ $\quad du = e^x dx$

$dv = \sin x\,dx$ $\quad v = -\cos x$

$\int u\,dv = -e^x \cos x + \int e^x \cos x\,dx$

$\int e^x \cos x\,dx = e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$

$2\int e^x \cos x\,dx = e^x(\sin x + \cos x) + C$

ex 5 $\int_0^1 \tan^{-1}x \, dx$

$u = \tan^{-1}x \quad du = \frac{1}{1+x^2}dx$

$dv = dx \quad v = x$

$$\int_{x=0}^{x=1} u\,dv = (\tan^{-1}x)x - \int_{x=0}^{x=1} \frac{x}{1+x^2}dx$$

$u = 1+x^2$

$du = 2x\,dx$

$$-\frac{1}{2}\int_0^1 \frac{1}{u}du$$

$$(\tan^{-1}x)x - \frac{1}{2}\ln|1+x^2| \Big|_0^1$$

$$\left(\frac{\pi}{4}\right)\cdot 1 - \frac{1}{2}\ln 2 + \cancel{\frac{1}{2}\ln 1}$$

$$\frac{\pi}{4} - \frac{1}{2}\ln 2$$

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$\int x\cos 3x\,dx$

let $u = x$ $\quad du = dx$

$dv = \cos 3x\,dx$ $\quad v = \frac{1}{3}\sin 3x$

$$\int u\,dv = \frac{1}{3}x\sin 3x - \int \frac{1}{3}\sin 3x\,dx$$

$$-\frac{1}{3}\left(\frac{1}{3}\right)\int \sin 3x\,(3dx) = -\frac{1}{9}(-\cos 3x) + C$$

$$= \frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C$$

HW:

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