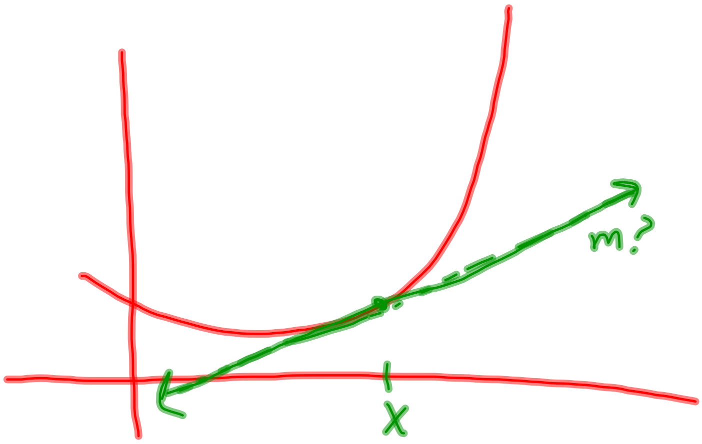
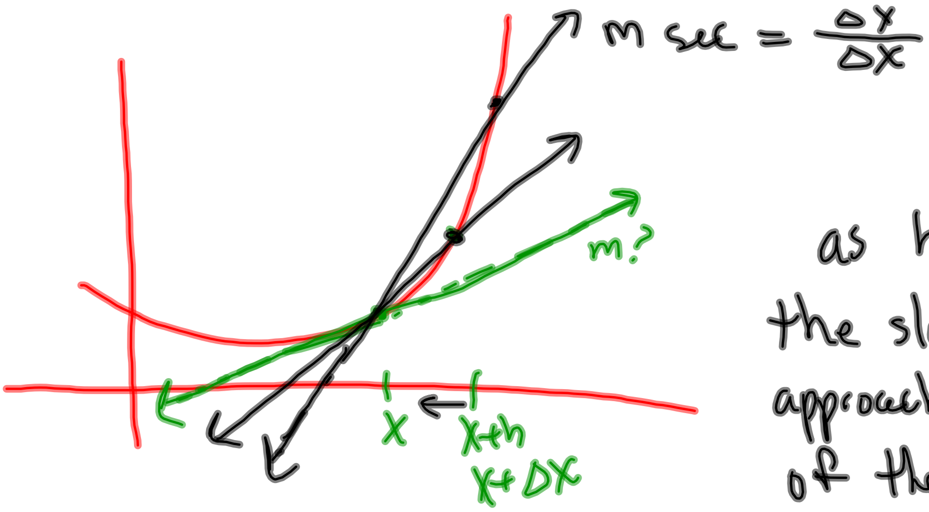
Limits

tangent
area
instantaneous speed

draw tangents
draw rectangles

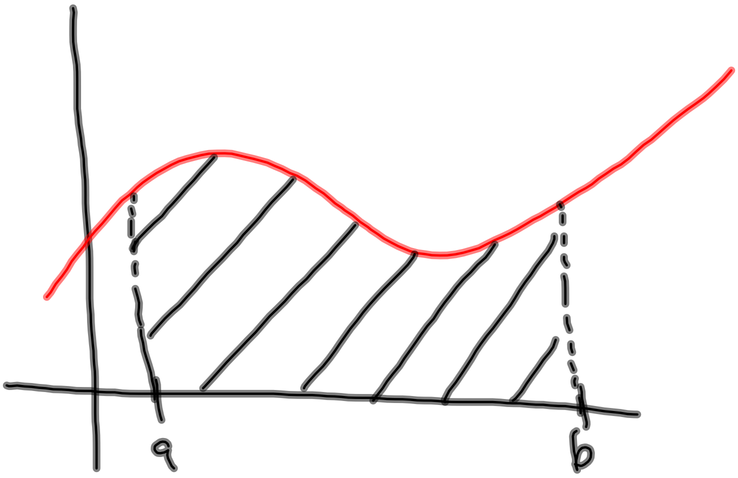
Slope of tan at x?



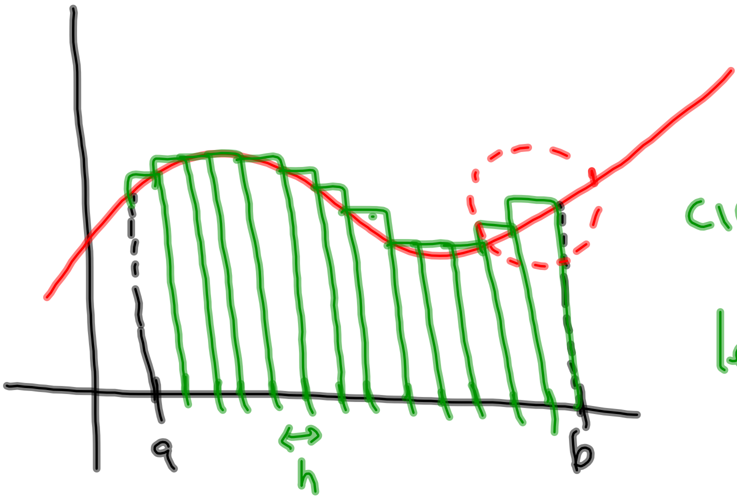


as $h \to 0$
the slope of secant
approaches the slope
of the tangent

Area under curve



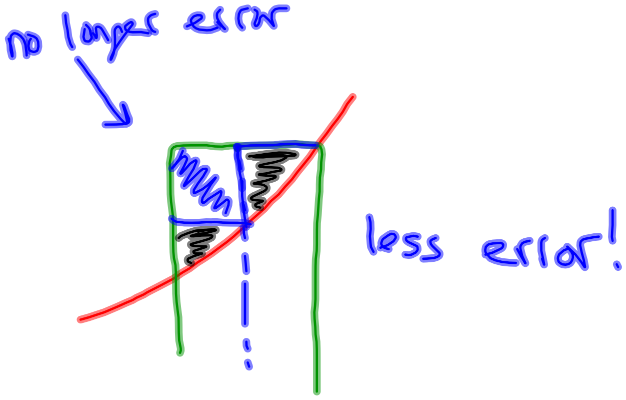
Area under curve



circumscribed rectangles

let $h \to 0$ error $\to 0$

$$h = \frac{b-a}{n}$$

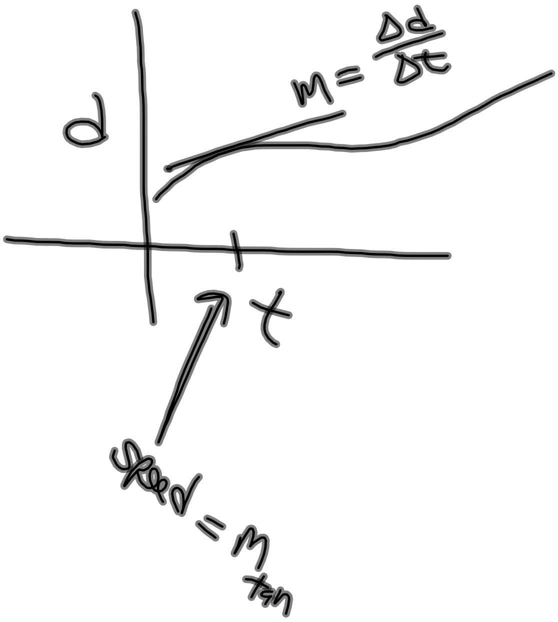


$n \to \infty$

$h \to 0$

error $\to 0$

$A_{rect} \to A_{under\ curve}$


d
$m = \frac{\Delta d}{\Delta t}$
t
$\text{speed} = m_{tan}$

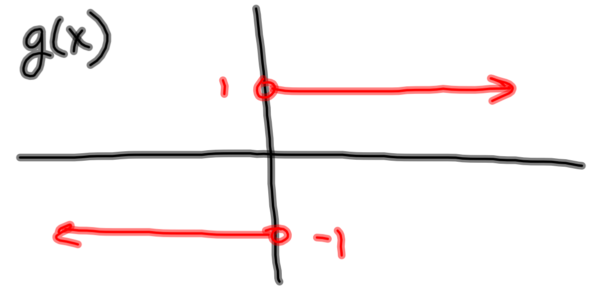
see fig 2.1.9 pg 115 – table of values

$$\lim_{x\to 2} f(x) = 3$$

3 pics p116, p117

$\lim_{x\to a} f(x) = f(a)$? not necessary

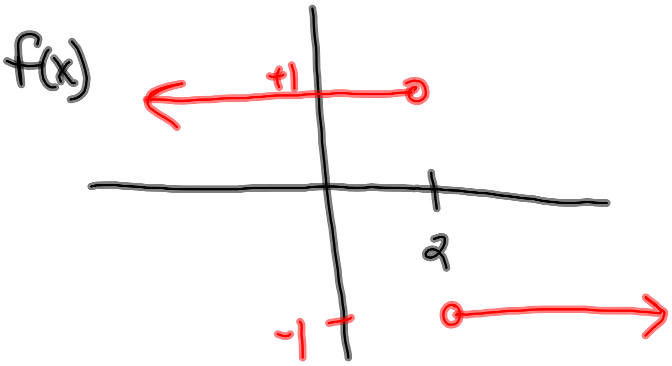
1-sided limits



$$\lim_{x\to 0^+} g(x) = 1$$

$$\lim_{x\to 0^-} g(x) = -1$$

$$\lim_{x\to 0} g(x) = \text{und}$$



$$\lim_{x\to 2^+} g(x) = -1$$

$$\lim_{x\to 2^-} g(x) = 1$$

$$\lim_{x\to 2} f(x) = \text{unde}$$

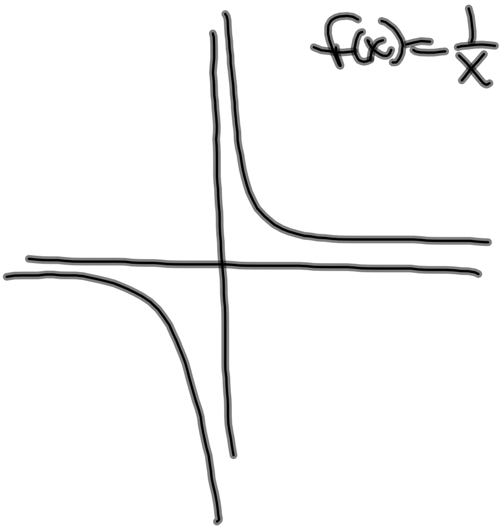
continuous curve def pg 120

smooth, no jumps, no holes

$f(x)$ is continuous at x_1

iff $\lim_{x \to x_1} f(x)$ exists an $= f(x_1)$

inf limits (vocab) pg 120



$\lim_{x\to 0^+} f(x) =$

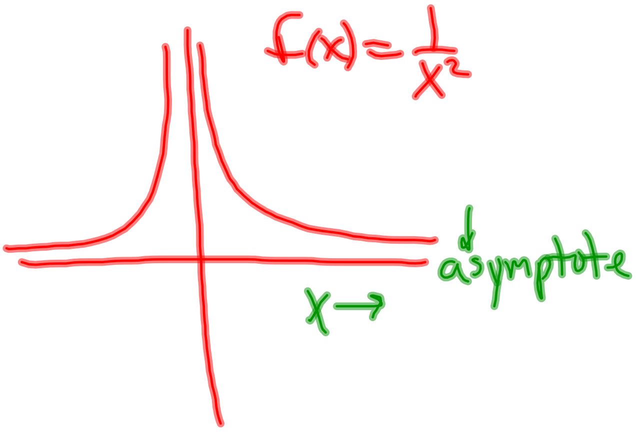
$f(x)$ "gets large without bound"

$\lim_{x\to 0^+} f(x) = \infty$

$\lim_{x\to 0^-} f(x) = -\infty$

limits at inf pg 122

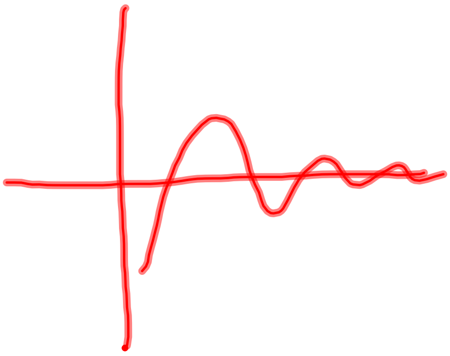
$f(x) = \frac{1}{x^2}$



$$\lim_{x \to \infty} f(x) = 0^+$$

"limit as x gets large without bound"

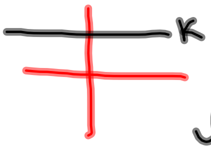
$$\lim_{x \to -\infty} f(x) = 0^-$$



computing limits p128
algebraic limits p 129
rational function limits p132 exs 5,6,7,8,11

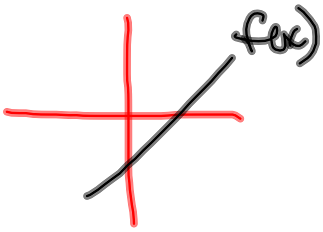
epsilon/delta sec 2.3

therom 2.2.1 pg 128



$f(x)=k$

$\lim_{x \to 0} f(x)=k$



$\lim_{x \to a} f(x)= f(a)$

$$\lim\left[f(x) \pm g(x)\right] = \lim f(x) \pm \lim g(x)$$

$$\lim\left[f(x) \times g(x)\right] = \left[\lim f(x)\right]\left[\lim g(x)\right]$$

for quotient $\lim g(x) \neq 0$

$$\lim \sqrt[n]{f(x)} = \sqrt[n]{\lim(f(x))} \leftarrow \boxed{\begin{array}{l} n \text{ is even} \\ \lim f(x) \geq 0 \end{array}}$$

pg 132 ex 5

$$\lim_{x\to 2} \frac{5x^3+4}{x-3} = \frac{5(8)+4}{-1} = -44$$

ex 6

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$$ ok because we dont care about f(2)

$= 4$

ex 8

$$\lim_{x \to 4^+} \frac{2-x}{(x-4)(x+2)} = \frac{-2}{(+)\,6} = -\infty$$

$$\lim_{x \to 4^-} \frac{2-x}{(x-4)(x+2)} = \frac{-2}{(-)\,6} = +\infty$$

$$\boxed{\lim_{x \to 4} \frac{2-x}{(x-4)(x+2)} \quad \text{d.n.e.}}$$

ex 11

a) $$\lim_{x\to\infty}\frac{3x+5}{6x-8}=\lim_{x\to\infty}\frac{3+\overset{\nearrow 0}{\boxed{\frac{5}{x}}}}{6-\overset{\nearrow 0}{\boxed{\frac{8}{x}}}}=\frac{3}{6}=\frac{1}{2}$$

b) $$\lim_{x\to-\infty}\frac{4x^2-x}{2x^3-5}=\lim_{x\to-\infty}\frac{\frac{4}{x}-\frac{1}{x^2}}{2-\frac{5}{x^3}}=\frac{0-0}{2-0}=0$$

c) $$\lim_{x\to\infty}\frac{3-2x^4}{x+1}=\lim_{x\to\infty}\frac{\frac{3}{x^4}-2}{\frac{1}{x^3}+\frac{1}{x^4}}=\frac{0-2}{0+0}=\frac{-2}{0}$$

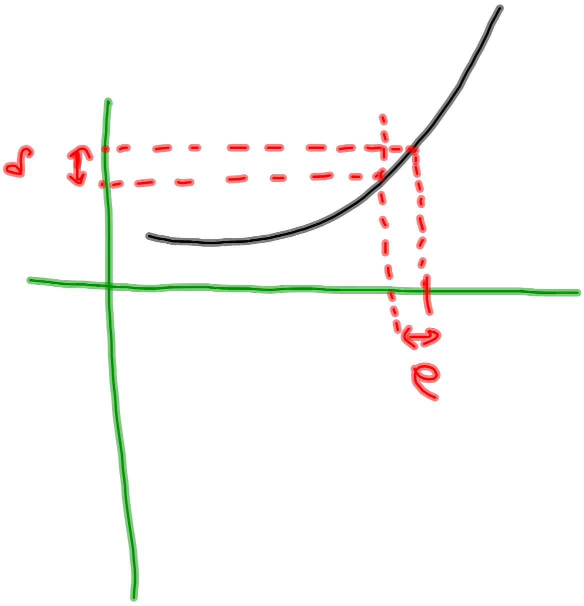
$$=-\infty$$

$$\lim_{x\to\infty} \frac{3x}{2x^2-6} = 0$$

$$\lim_{x\to\infty} \frac{6x^3+4}{5x^3-2x^2+1} = \frac{6}{5}$$

$$\lim_{x\to\infty} \frac{3x^4}{7x+2} = \infty$$

epsilon-delta (sec 2.3)



pg 137
#s 5,25,39,55