Integration using **Partial Fractions**

this is why you spent so much time on those ugly rational functions in Algebra I and Algebra II!

$$\frac{2}{x-4}+\frac{3}{x+1}=\frac{2(x+1)+3(x-4)}{(x-4)(x+1)}=\frac{2x+2+3x-12}{}$$

$$=\frac{5x-10}{x^2-3x-4}$$

$$\left[\frac{5x-10}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}\right](x+1)(x-4)$$

$$5x-10 = A(x+1) + B(x-4)$$

$$5 = A+B$$

$$-10 = A-4B$$

$$A = 5-B \Rightarrow -10 = 5-B-4B$$

$$-10 = 5-5B$$

$$3 = B \Rightarrow A = 2$$

linear denominator $\Rightarrow$ constant numerator

$$\frac{A}{(x-4)}$$

quadratic denominator $\Rightarrow$ linear numerator

$$\frac{Bx+C}{(x^2+1)}$$

repeated factors must be used twice

$(x-2)^2$ $\qquad$ $\frac{A}{x-2} + \frac{B}{(x-2)^2}$

ex) $\int \frac{dx}{x^2-x-2}$

$$\left[\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}\right](x-1)(x+2)$$

$1 = (x+2)A + (x-1)B$

$0 = A+B$

$1 = 2A-B$

$1 = 2(-B)-B$

$1 = 2B$

$B = \frac{1}{2} \Rightarrow A = \frac{1}{2}$

$$\frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{2}}{x+2}$$

↓ integrate

ex $\int \frac{2x+4}{x^3-2x^2}\,dx$

$$\frac{2x+4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} \qquad \times x^2(x-2)$$

$$2x+4 = A(x)(x-2) + B(x-2) + Cx^2$$
$$= Ax^2 - 2Ax + Bx - 2B + Cx^2$$

$0 = A + C$

$2 = -2A + B$ $\hookrightarrow 2 = -2A - 2 \Rightarrow A = -2 \Rightarrow C = 2$

$4 = -2B \Rightarrow B = -2$

$$-\frac{2}{x} - \frac{2}{x^2} + \frac{2}{x-2}$$ ↘ integrate

ex3/ $\int \frac{x^2+x-2}{3x^3-x^2+3x-1}\,dx$

$(3x^3-x^2)+(3x-1)$

$x^2(3x-1)+1(3x-1)$

$(3x-1)(x^2+1)$

$\frac{x^2+x-2}{(3x-1)(x^2+1)} = \frac{A}{3x-1} + \frac{Bx+C}{x^2+1}$ $\times (3x-1)(x^2+1)$

$x^2+x-2 = (x^2+1)A + (3x-1)(Bx+C)$

$= Ax^2+A+3Bx^2+3Cx-Bx-C$

$1 = A+3B$

$1 = 3C-B$ $\times 3$

$-2 = A-C$ $\times 3$

$-5 = 3A-B$

$1 = A+3B$

$-4 = 10A$

$A = -\frac{2}{5}$ $B = \frac{4}{5}$ $1 = \frac{3}{5}$

$$\frac{-\frac{7}{5}}{3x-1} + \frac{\frac{4}{5}x+\frac{3}{5}}{x^2+1}$$

$\ln|3x-1|$

$$\Rightarrow \frac{\frac{4}{5}x}{x^2+1} + \frac{\frac{3}{5}}{x^2+1}$$

$$\downarrow$$

$$\ln|x^2+1| + \tan^{-1}(x)$$

#16 p 543

$$\int \frac{x^2-4}{x-1}\,dx$$

$$\begin{array}{r} x+1 \\ x-1 \overline{)\,x^2+0x-4} \\ (-)\,x^2-x \\ \hline x-4 \\ (-)\,x-1 \\ \hline -3 \end{array}$$

$$\int \left(x+1-\frac{3}{x-1}\right)dx$$

$$\frac{x^2}{2}+x-3\ln|x-1|+C$$

HW:

page 542: 9,15,21