

"function" definition

function tests

"relation" definition

function notation: straight lines

function notation: anything

implicit/explicit definition

implicit functions

Relation - set of ordered pairs

function - relation where $\forall x \in E$
at most 1 y

(must pass "vertical" line test)

linear functions

$$ax + by + c = 0$$

$$ax + by = c$$

$$y = mx + b$$

Y "is a function of" X

Y depends on X

X = independent variable

Y = dependent variable

$$3x + 8y - 4 = 0$$

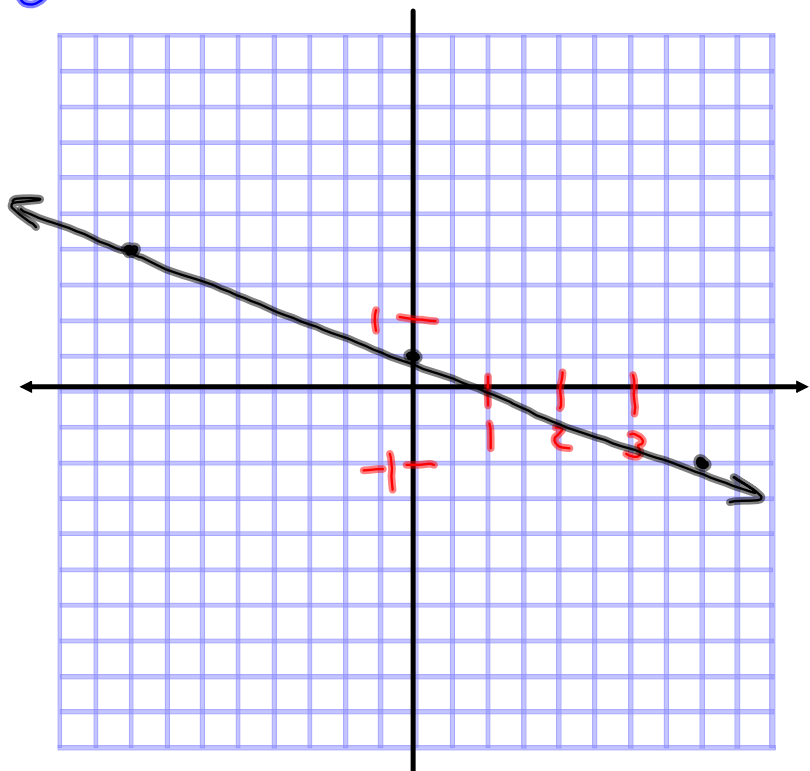
$$3x + 8y - 4 = 0$$

$$8y = -3x + 4$$

$$y = -\frac{3}{8}x + \frac{1}{2}$$

$$m = -\frac{3}{8}$$

$$b = \frac{1}{2}$$



$$3x + 8y - 4 = 0$$

∴

$$y = -\frac{3}{8}x + \frac{1}{2}$$

$$y = f(x) \quad \text{"f of x"}$$

$$f(x) = 3x + 6$$

$$\begin{aligned} f(-2) &= 3(-2) + 6 \\ &= -6 + 6 \\ &= 0 \end{aligned}$$

$$\underline{(-2, 0)}$$

$$y = 3x + 6$$

$$y = 3(-2) + 6$$

$$y = -6 + 6$$

$$y = 0$$

$$\underline{(-2, 0)}$$

$$f(x) = -2x^2 + x - 3$$

$$\begin{aligned} f(1) &= -2(1)^2 + (1) - 3 \\ &= -2 + 1 - 3 \\ &= -4 \end{aligned}$$

$$(1, -4)$$

$$y = -2x^2 + x - 3$$

~~magic~~

$$f(x) = \sin x$$

$$f(1) = .8414 \quad (\text{radians})$$

$$f(\pi) = 0$$

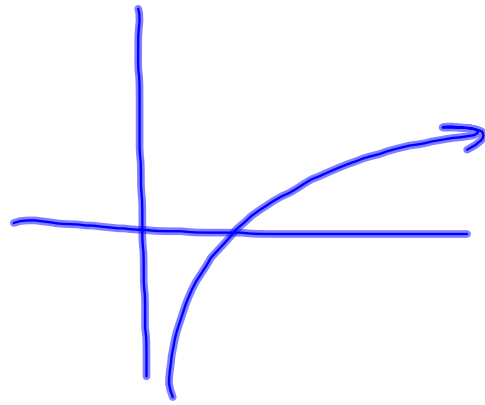
Domain: possible x values "input"
Range: possible y values "output"

$$f(x) = 3x \quad D: \{x: x \in \mathbb{R}\}$$
$$R: \{y: y \in \mathbb{R}\}$$

$$y = f(x) = \ln x$$

$$D: \{x: x > 0\}$$

$$R: \{y: y \in \mathbb{R}\}$$



$$y = f(x) = \frac{3}{x^2 - 4}$$

$$D: \{x: x \neq \pm 2\}$$

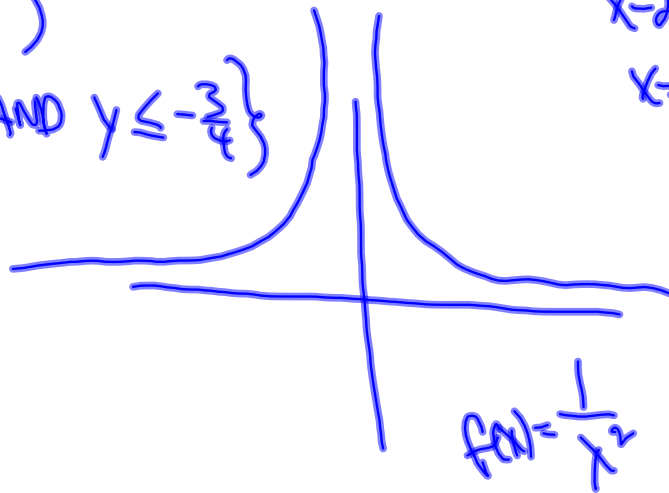
$$R: \{y: y > 0 \text{ AND } y \leq -\frac{3}{4}\}$$

$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$

$$x+2=0$$

$$x-2=0$$

$$x = \pm 2$$



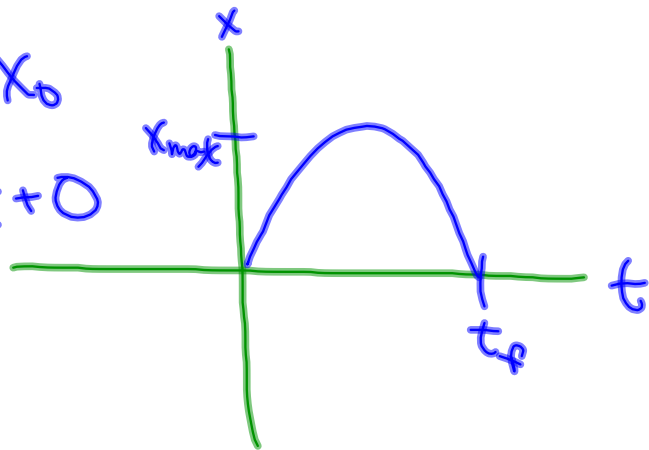
Physical restraints on Domain, Range

$$x = \frac{1}{2}gt^2 + v_0t + x_0$$

$$x = \frac{1}{2}(-9.8)t^2 + (+10)t + 0$$

$$D: \{t: 0 \leq t \leq t_f\}$$

$$R: \{x: 0 \leq x \leq x_{\max}\}$$



An object thrown into the air only follows the given function (position as a function of time) while it is in the air

Absolute value

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$y = |x-3|$$

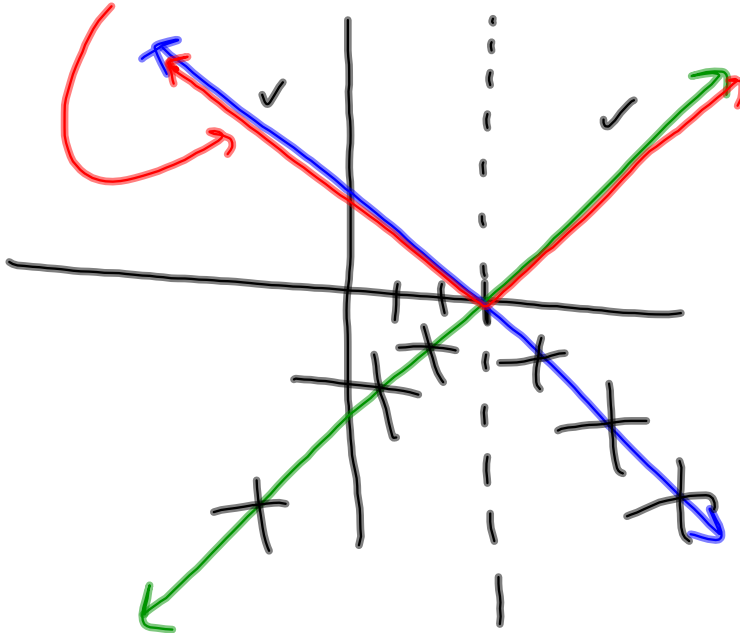
$$y = x-3$$

$$x-3 \geq 0 \Rightarrow x \geq 3$$

$$y = -(x-3) = -x+3$$

$$x-3 < 0$$

$$x < 3$$



become an expert at graphing function
and adjusting the view on your graphing
calculator. **do it now!** :)

algebraic operations on functions: **Notation**

$$f(x) - g(x) = (f - g)(x)$$

$$f(x) + g(x) = (f + g)(x)$$

$$f(x) * g(x) = (f * g)(x)$$

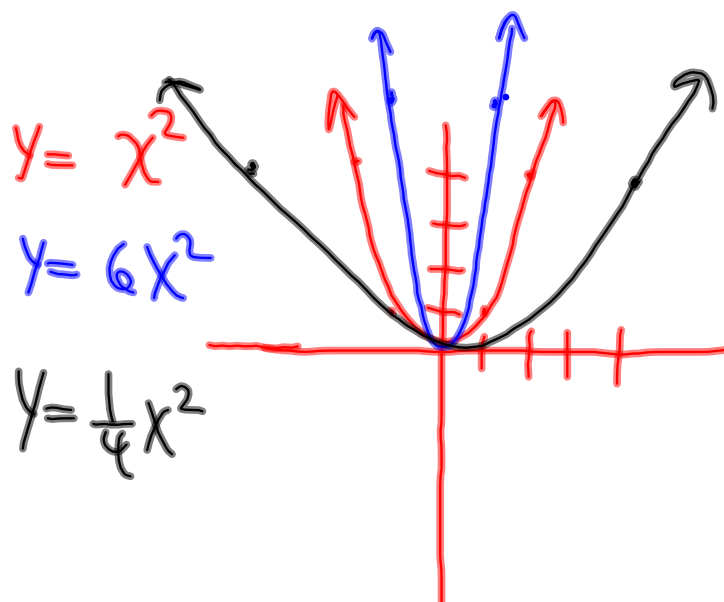
$$f(x) \div g(x) = (f / g)(x) \quad g(x) \neq 0$$

$$f(x) = 3x + 2 \quad g(x) = x$$

$$(f - g)(x) = 3x + 2 - x = 2x + 2$$

$$(f * g)(x) = (3x + 2)(x) = 3x^2 + 2x$$

stretching and compressing graphs
(pics on pg 48-49)



Composition of functions

$$f(g(x)) = (f \circ g)(x)$$

$$g(f(x)) = (g \circ f)(x)$$

$$f(x) = 3x + 2 \quad g(x) = x - 1$$

$$(f \circ g)(x) = f(g(x)) = f(x-1) \\ = 3(x-1) + 2 = 3x - 3 + 2 = 3x - 1$$

$$(g \circ f)(x) = g(3x+2) = (3x+2) - 1 \\ = 3x + 1$$

Translations (pg 54 ex9,10)

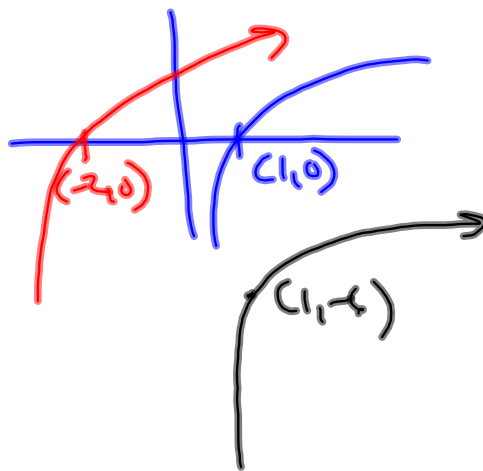
$$f(x) \Rightarrow f(x) + c \quad \text{shift } c \text{ units} \\ \text{on } y \text{ axis}$$

$$f(x) \Rightarrow f(x-c) \quad \text{shift } c \text{ units} \\ \text{on } x \text{ axis}$$

$$y = \ln(x)$$

$$y = \ln(x) - 6$$

$$y = \ln(x+3)$$
$$\ln(x - (-3))$$



Reflections (pg 56)

$$f(-x)$$

reflection in y-axis

$$-f(x)$$

reflection in x-axis

$$(-y)$$

Families of functions

linear

$$y = x$$

$f(x) = x^n$

$$y = x^2 \quad // \quad y = x^3$$

$f(x) = x^{-n}$

$f(x) = x^{1/n}$

$$y = \sqrt{x}$$

rational

$$y = \frac{x+1}{x-1}$$

ratio of polynomials

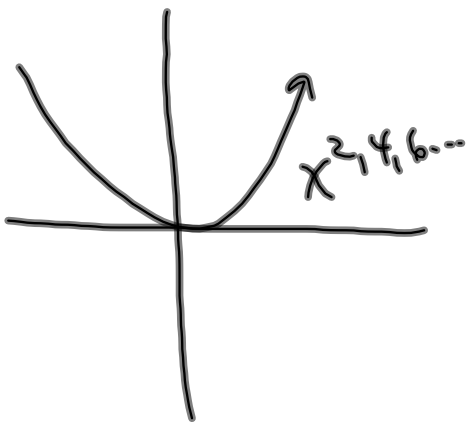
trigonometric

$\sin x, \text{ etc}$

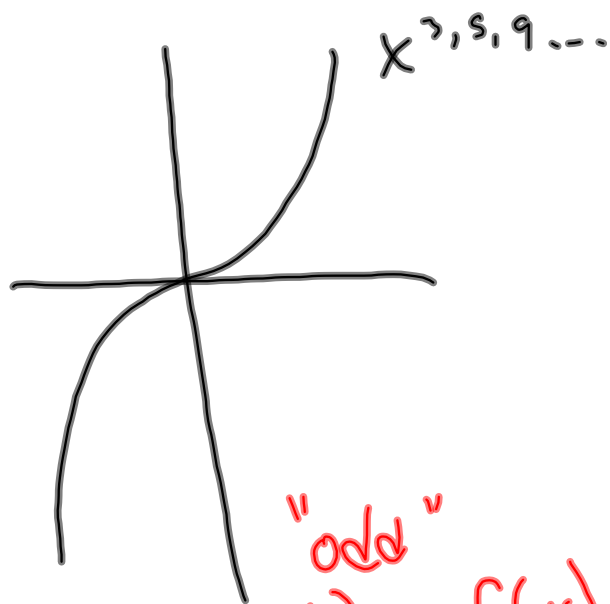
exponential and logarithmic

$\ln x, e^x$

$$f(x) = x^n = a(x-h)^n + k$$

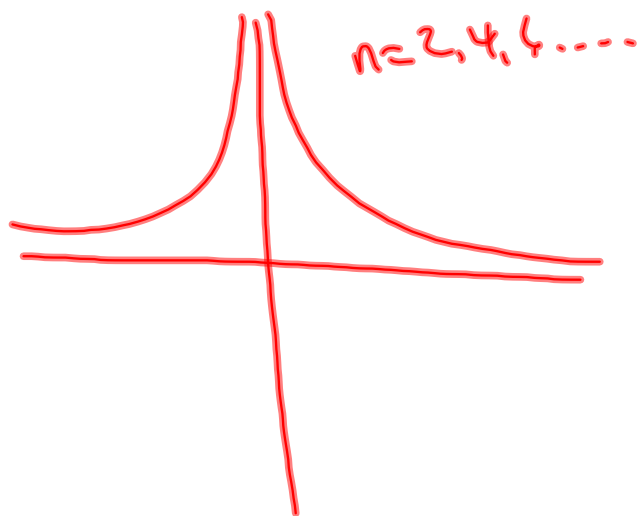
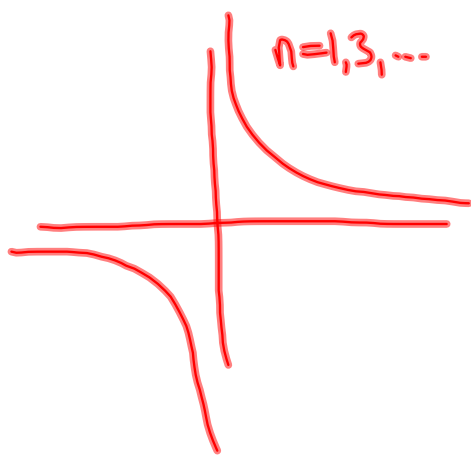


"even"
 $f(x) = f(-x)$

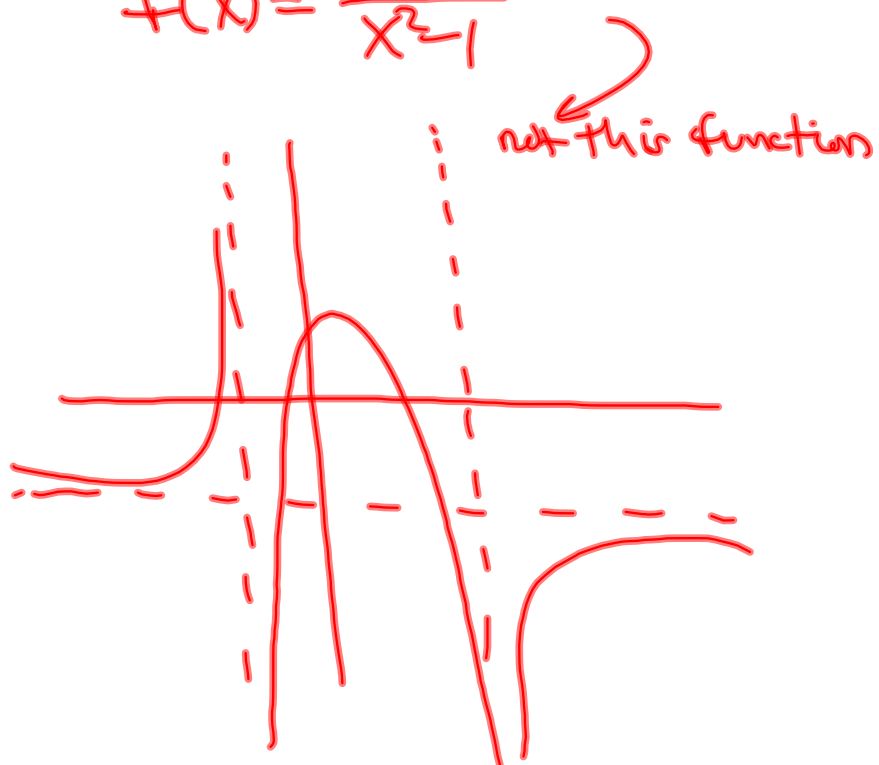


"odd"
 $f(x) = -f(-x)$

$$f(x) = x^{-n}$$

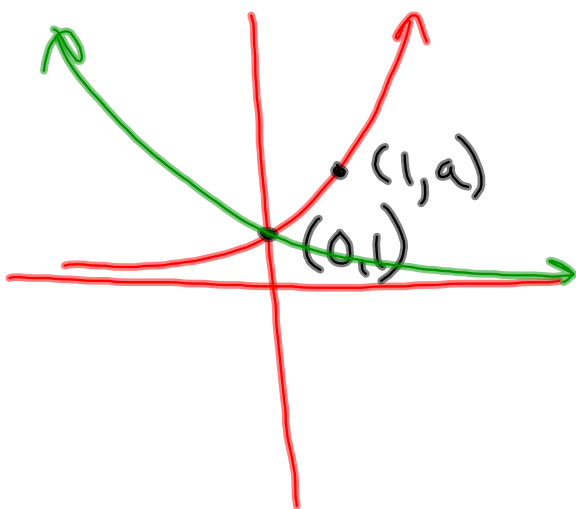


$$f(x) = \frac{x^2 + 3}{x^2 - 1}$$



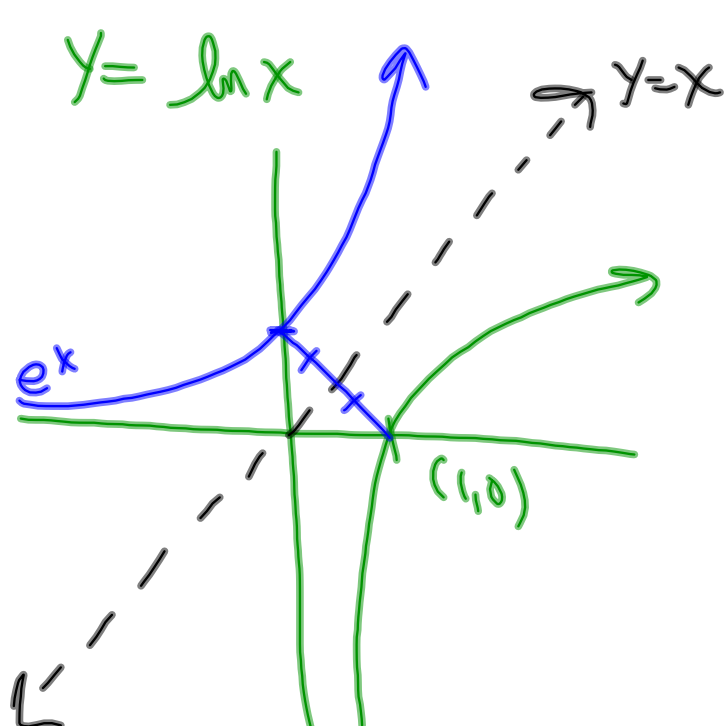
$$y = e^x$$

$$y = a^x$$



$$a < 1$$

2.718281828459045



reflections in
 $y=x$ are
 inverse functions

$$f(g(x)) = x$$

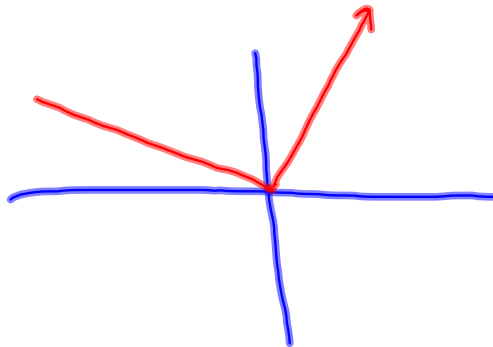
$$g(f(x)) = x$$

$$\ln(e^x) = x$$

$$e^{\ln x} = x$$

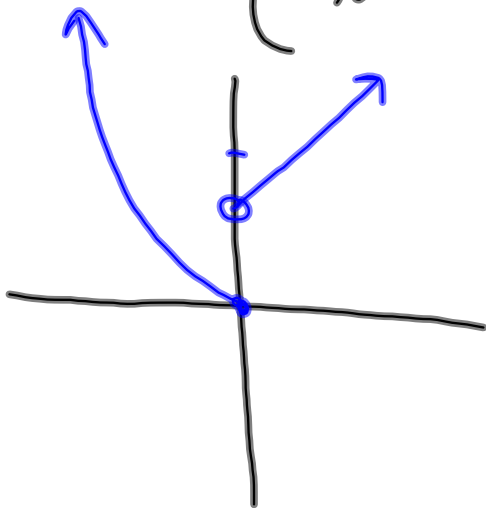
Piecewise functions

$$f(x) = \begin{cases} 3x & \text{if } x > 0 \\ -2x & \text{if } x \leq 0 \end{cases}$$



$$\begin{aligned} f(-2) &= 4 \\ f(2) &= 6 \\ f(0) &= 0 \end{aligned}$$

$$f(x) = \begin{cases} 2x+2 & x > 0 \\ x^2 & x \leq 0 \end{cases}$$



Implicit: I expected you to do a better job
Explicit: you are a moron

Explicit $f(x) = 3x + 6$
 $y = 3x + 6$

Implicit $xy = 3$
 $xy + y = 3$

$y = f(x)$

homework 7-8-13 (4th edition)

sec2-1 (pg81) 4,9,11,25,37,43,51,61

sec2-2(pg90) 1,3,7,15,21,34

sec2-3(pg102) 11,36,42,47,57

$$\textcircled{4} \quad g(x) = \begin{cases} \sqrt{x+1} & x \geq -1 \\ 3 & x < -1 \end{cases}$$

$$g(0) =$$

$$g(-1) =$$

$$g(-1.1) =$$

$$g(\pi) =$$

$$g(3) =$$

$$g(t^2 - 1) =$$

$$\textcircled{4} \quad g(x) = \begin{cases} \sqrt{x+1} & x \geq -1 \\ 3 & x < -1 \end{cases}$$

$$g(0) = 1$$

$$g(-1) = 0$$

$$g(-1.1) = 3$$

$$g(\pi) = \sqrt{\pi+1}$$

$$g(3) = 2$$

$$g(t^2-1) = \sqrt{t^2-1+1} = |t|$$

⑨

$$h(x) = \sqrt{\frac{x-1}{x+2}}$$

find Domain

⑨ $h(x) = \sqrt{\frac{x-1}{x+2}}$ find Domain

$$\frac{x-1}{x+2} > 0$$

$$x-1 > 0$$

$$x > 1$$

$$(x+2 > 0)$$

$$x > -2$$

"AND" = conjunction

$$x-1 < 0$$

$$x < 1$$

$$(x+2 < 0)$$

$$x < -2$$

"OR"

$$D: \{x: x < -2 \text{ or } x > 1\}$$

.."

⑪ $\phi(x) = \frac{x}{|x|+1}$ find Domain

⑪ $f(x) = \frac{x}{|x|+1}$ find Domain

$$|x|+1=0 ?$$

$$|x|=-1 \quad \times$$

$$D: \{x: x \in \mathbb{R}\}$$

(#25) $g(x) = \sqrt{4-x^2}$

$$4-x^2 \geq 0$$

$$(2-x)(2+x) \geq 0$$

either

$$2-x \geq 0 \text{ AND } 2+x \geq 0$$

$$-x \geq -2$$

$$x \leq 2$$

$$-2 \leq x \leq 2$$

(OR)

$$2-x < 0 \text{ AND } 2+x < 0$$

$$-x < -2$$

$$x > 2$$

$$x \leq -2$$

\emptyset

$$D: \{x: -2 \leq x \leq 2\}$$

#25

$$g(x) = \sqrt{4-x^2}$$

$$R: \{y: 0 \leq y \leq 2\}$$

$$y = 4 - x^2 \text{ has vertex at } \\ \frac{-b}{2a} = \frac{0}{2(-1)} = 0 \quad V(0, 4)$$

y is concave down
($a < 0$)

$\therefore y$ has max at $x=0$

$\therefore g$ has max at $x=0$

$$g(0) = 2$$

$$\textcircled{37} \quad f(x) = |x| + 3x + 1$$

$$f(x) = \begin{cases} 4x + 1 & \text{if } x \geq 0 \\ 2x + 1 & \text{if } x < 0 \end{cases}$$

$$\text{if } x < 0 \quad |x| = -x$$

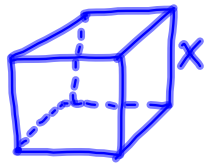
$$\textcircled{43} \quad f(x) = x^2 + 5 \quad a = 7$$

$$f(x) = 7 = x^2 + 5$$

$$2 = x^2$$

$$x = \pm\sqrt{2}$$

51



6 sides $A_s = x^2$
 $S.A. = 6x^2$
 $S(x) = 6x^2$

$$V = A_b h = x^2 \cdot x = x^3$$

$$\frac{S}{6} = \frac{6x^2}{6}$$
$$\left(\frac{S}{6}\right)^{3/2} = (x^2)^{3/2}$$

$$\left(\frac{S}{6}\right)^{3/2} = x^3 = V$$

$$\frac{S}{6} = V^{2/3}$$

$$S = 6V^{2/3}$$

$$S(V) = 6V^{2/3}$$

61

$$\frac{1 - \frac{1}{x}}{1 + \frac{1}{x}} \left(\frac{x}{x} \right) \stackrel{?}{=} \frac{x-1}{x+1} \quad ?$$

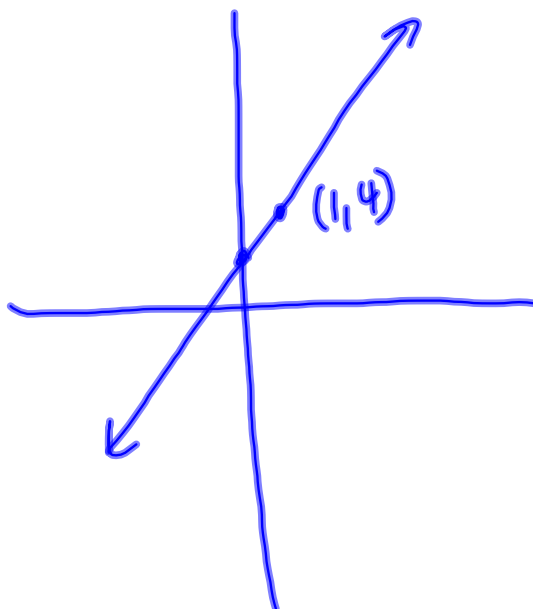
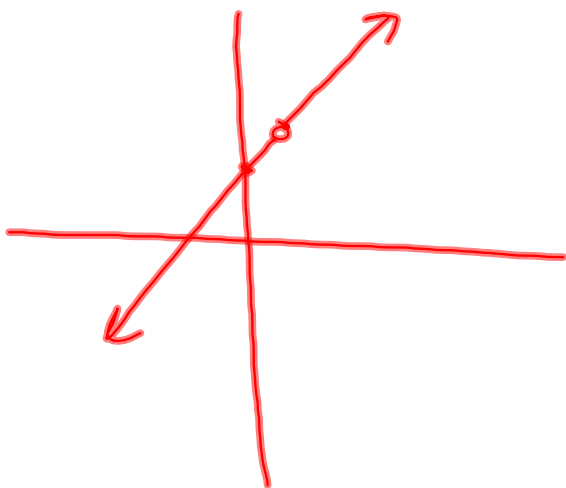
always true?

NO

$$\frac{1 - \frac{1}{0}}{1 + \frac{1}{0}}$$

$$\frac{0-1}{0+1} = -1$$

$$y = \frac{(3x+1)(x-1)}{(x-1)} \stackrel{?}{=} 3x+1$$



$$\textcircled{1} f(x) = x^2 + 1$$

$$f(t) =$$

$$f(t+2) =$$

$$f(x+2) =$$

$$f\left(\frac{1}{x}\right) =$$

$$f(x+h) =$$

$$f(-x) =$$

$$f(\sqrt{x}) =$$

$$f(3x) =$$

$$\textcircled{1} f(x) = x^2 + 1$$

$$f(t) = t^2 + 1$$

$$f(t+2) = (t+2)^2 + 1 = t^2 + 4t + 4 + 1 = t^2 + 4t + 5$$

$$f(x+2) = (x+2)^2 + 1 = x^2 + 4x + 5$$

$$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 + 1 = \frac{1}{x^2} + 1$$

$$f(x+h) = (x+h)^2 + 1 = x^2 + 2hx + h^2 + 1$$

$$f(-x) = (-x)^2 + 1 = x^2 + 1$$

$$f(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1 \quad x \geq 0$$

$$f(3x) = (3x)^2 + 1 = 9x^2 + 1$$

$$\textcircled{3} \quad f(1)=4 \quad f(2)=5 \quad g(1)=3 \quad g(2)=-1$$

$$\textcircled{3} \quad f(-1)=4 \quad f(2)=5 \quad g(-1)=3 \quad g(2)=-1$$

$$(f-g)(-1) = f(-1) - g(-1) = 4 - (3) = 1$$

$$(f/g)(2) = \frac{5}{-1} = -5$$

$$(f \cdot g)(-1) = 4(3) = 12$$

$$(f \circ g)(2) = f(g(2)) = f(-1) = 4$$

⑦

$$\textcircled{15} \quad h(x) = 2x - 5$$

$$h \circ h =$$

$$h^2 =$$

$$\textcircled{15} \quad h(x) = 2x - 5$$

$$h \circ h = h(h(x)) = h(2x - 5) = 2(2x - 5) - 5 \\ = 4x - 10 - 5 = 4x - 15$$

$$h^2 = (h(x))^2 = (2x - 5)^2 \\ = 4x^2 - 20x + 25$$

$$\textcircled{21} \quad f(x) = \sqrt{x+2}$$

$$g(x) = \sqrt{x}$$

$$h(x) = x+2$$

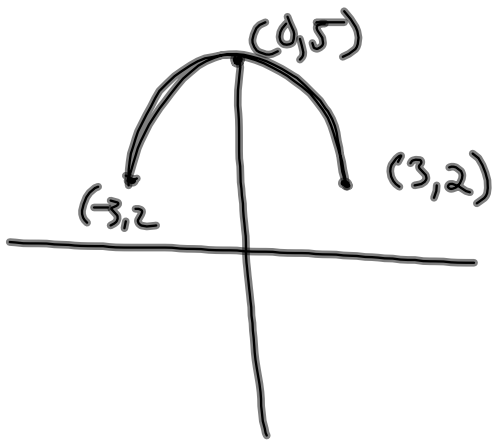
$$f(x) = g \circ h = g(h(x)) = g(x+2) = \sqrt{x+2}$$

$$\textcircled{34} \quad f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

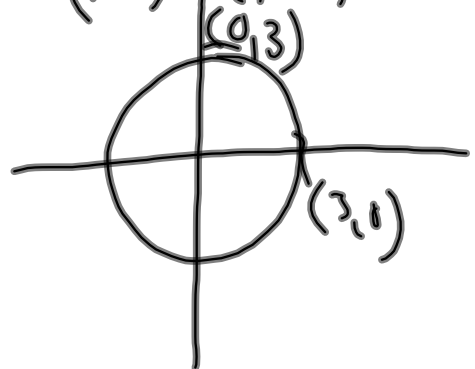
$$(f \circ f)(\pi)$$

$$= f(f(\pi)) = f(0) = 1$$

⑪ $f(x) = 2 + \sqrt{9 - x^2}$



$$y = \sqrt{9 - x^2}$$
$$y^2 = 9 - x^2$$
$$x^2 + y^2 = 9$$
$$(x-h)^2 + (y-k)^2 = r^2$$



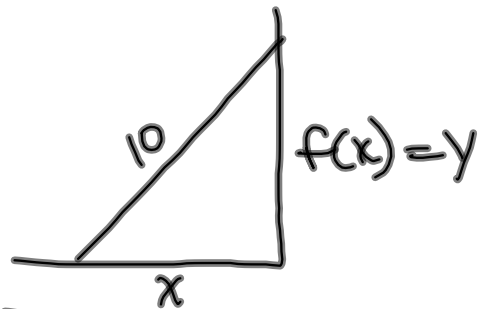
(36)

$$y^2 + x^2 = 10^2$$

$$y^2 = 10^2 - x^2$$

$$f(x) = y = \sqrt{100 - x^2}$$

$$0 \leq x \leq 10$$

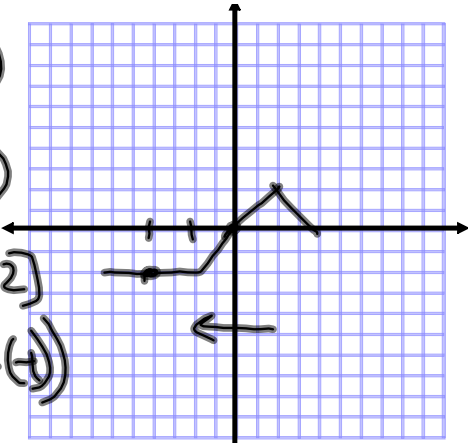


42

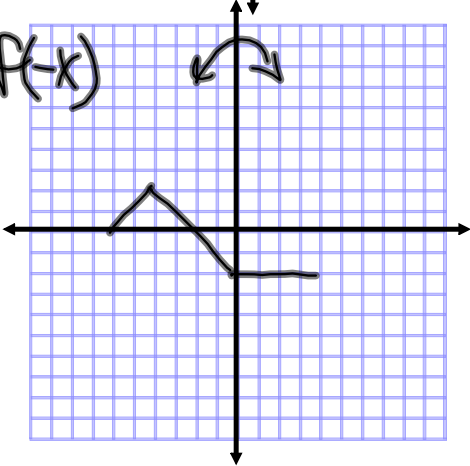
$f(x)$

$[-2, 2]$

$f(x-h)$

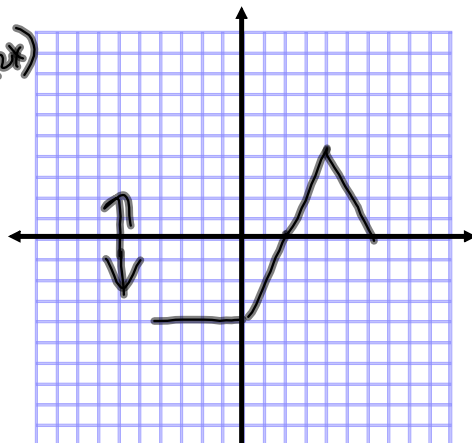


$y=f(-x)$

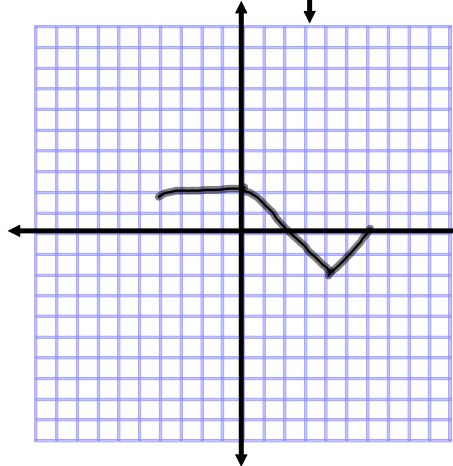


$f(x)$

y-axis stretch



$y=-f(x)$



④7) $f(-x) = f(x)$ even
 $f(-x) = -f(x)$ odd

a) even

b) odd

c) $f(x) = |x|$
even

$$f(-3) = |-3| = 3 \quad f(3) = |3| = 3$$

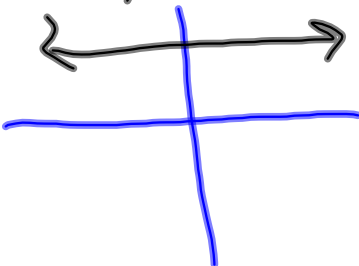
d) $f(x) = x+1$
neither

$$f(1) = 2 \quad f(-1) = 0 \quad -f(1) = -2$$

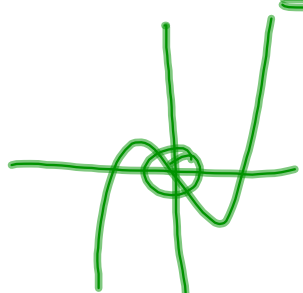
④7 $f(-x) = f(x)$ even
 $f(-x) = -f(x)$ odd

d) $f(x) = \frac{x^5 - x}{1+x^2} = \frac{x(x^4-1)}{1+x^2} = \frac{x(x^2-1)(x^2+1)}{1+x^2}$
 odd

e) $f(x) = 2$
 even



$= x^3 - x$



(57)

a) $4x + 2y = -8$ $y =$ both
 $x =$

b) $x^2 y^3 = 1$ $y^3 = \frac{1}{x^2}$ $y = \frac{1}{x^{2/3}}$

c) $x^2 + y^2 = 1$ $x^2 = \frac{1}{y^3}$ $x = \frac{1}{y^{3/2}}$ not a function
 (circle)
 neither

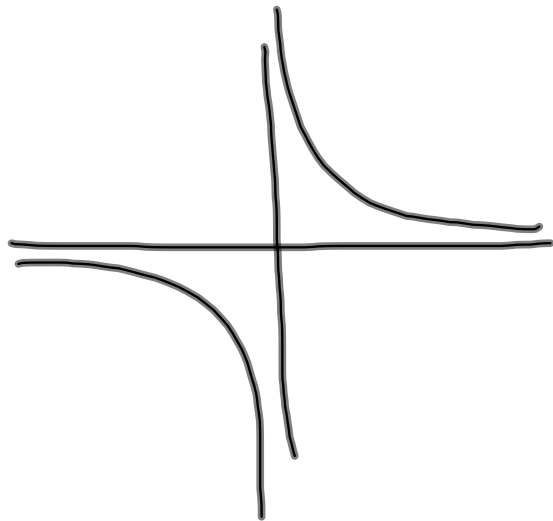
$$d) \frac{xy}{1-xy} = 1$$

$$xy = 1 - xy$$

$$2xy = 1$$

$$xy = \frac{1}{2}$$

$$y = \frac{1}{2x} \quad \text{or} \quad x = \frac{1}{2y}$$



slope of a straight line

Defn, examples, computation

slope of a curve...

 slope of a secant line

 limit(intuitive) as $h(\Delta x)$ gets small

 slope of tangent to curve

$$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

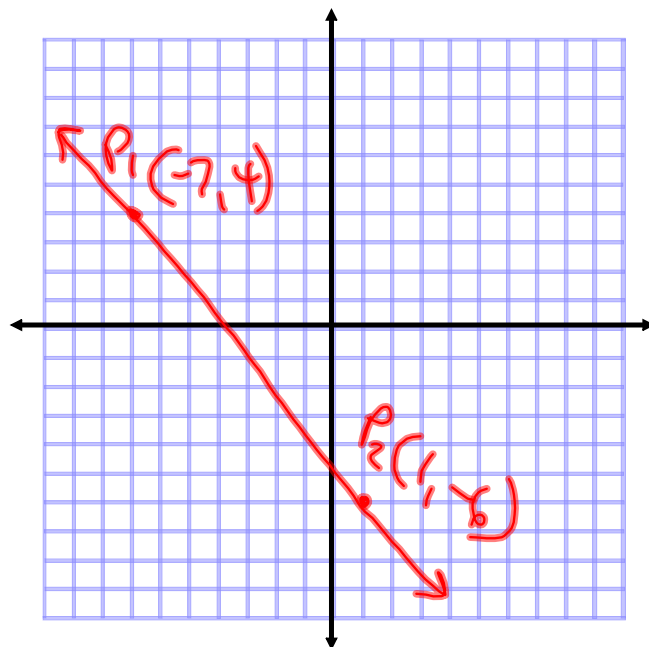
$$m = \frac{\Delta y}{\Delta x}$$

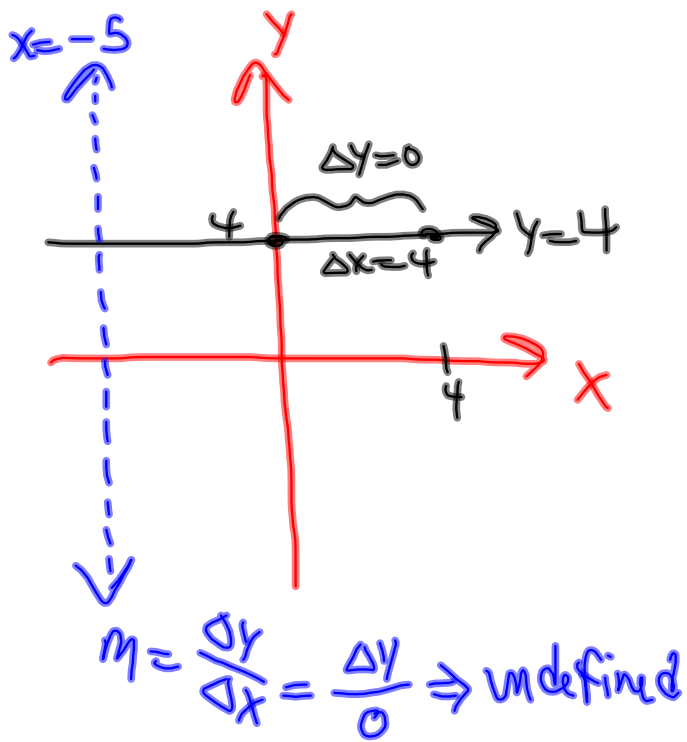
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-6 - 4}{1 - (-7)}$$

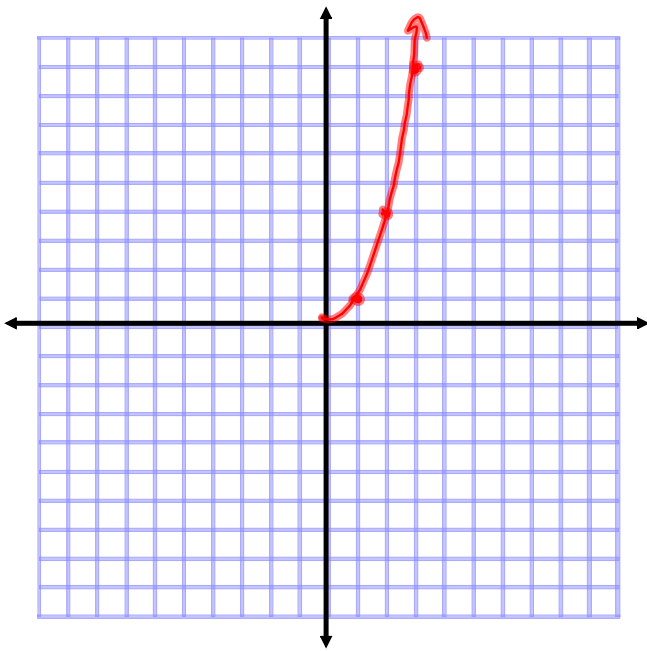
$$m = \frac{-10}{8}$$

$$m = -\frac{5}{4}$$

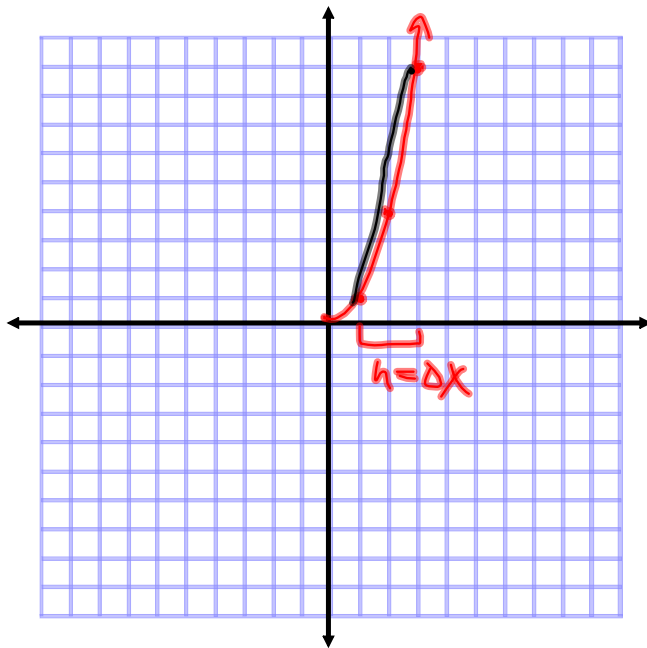




$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta y}{4} = \frac{0}{4} = 0$$



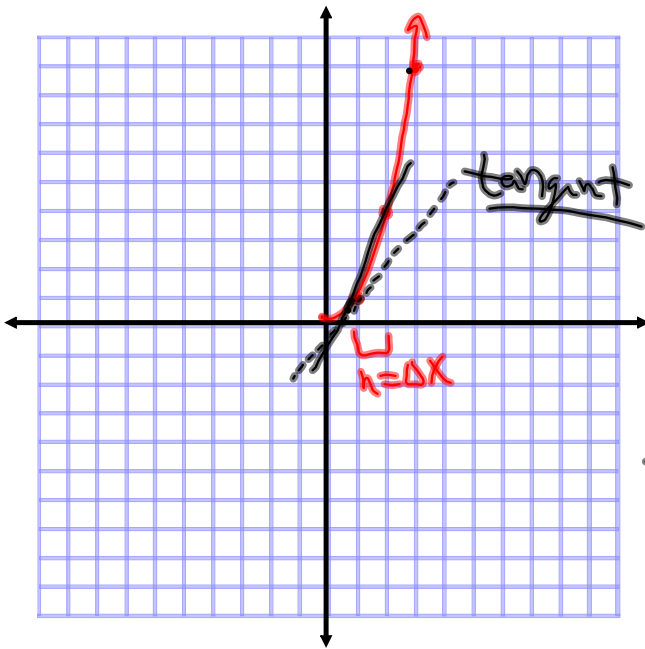
Slope??
 $(1,1)$, $(3,9)$



Slope??
 $(1,1)$, $(3,9)$

Secant line
(intersects graph at
2 points)

$$M_{\text{sec}} = \frac{8}{2} = 4$$



slope??

$(1, 1), (2, 4)$

$$m_{\text{sec}} = \frac{3}{1} = 3$$

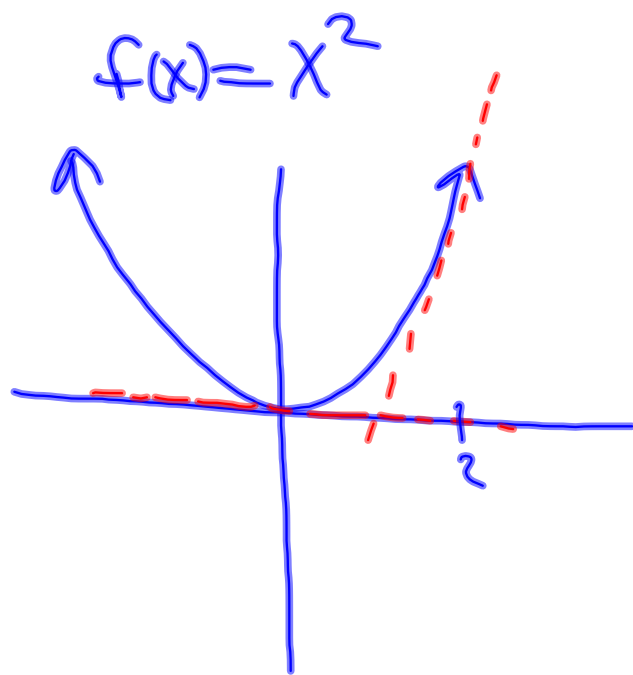
tangent intersects graph at exactly 1 point

for a "curve" $m = m_{\text{tan}}$

so m changes as x changes

so slope of curve is a function of x

(slope of straight line is constant)



$$M_f = 2x$$

$$M_{\text{tan}}(x=0) = 0$$

$$M_{\text{tan}}(x=2) = 4$$

$$M_{\text{tan}}(x=7) = 14 \text{ (steep)}$$

Velocity and Speed

speed is distance over time

velocity is a) speed in a particular direction

 b) displacement over time

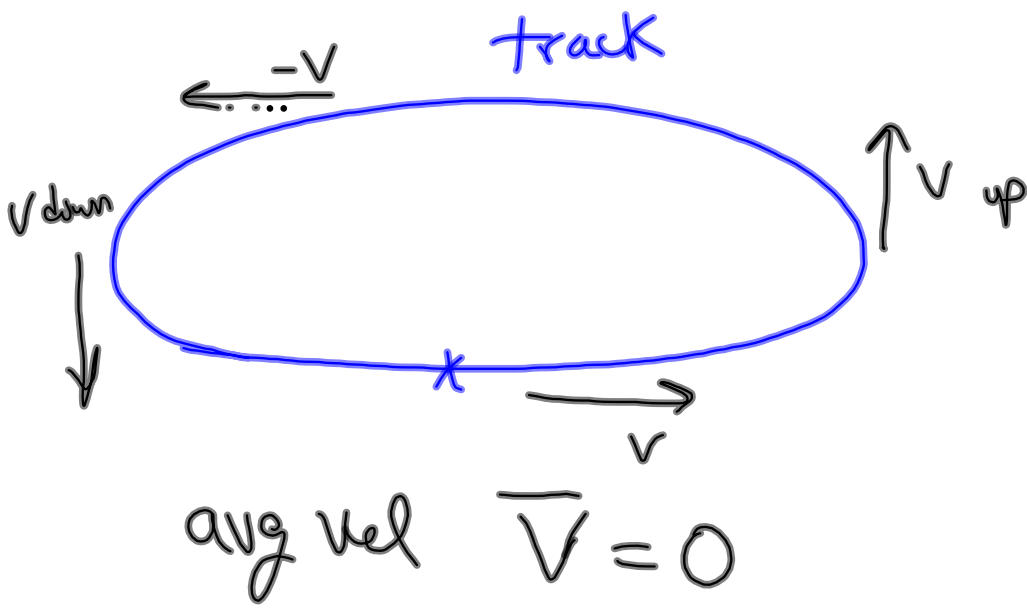
$\frac{1}{4}$ mile in 56 sec

$$\text{Speed} = \frac{\text{dist}}{\text{time}}$$

$$= \frac{.25 \text{ mi}}{.0156 \text{ hr}} = 16.1 \text{ mi/hr}$$

$$56 \text{ s} \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) = .0156$$

$$\text{Avg Velocity} = \frac{\text{disp}}{\text{time}} = \frac{0}{.0156 \text{ hr}} = 0 \text{ mi/hr}$$

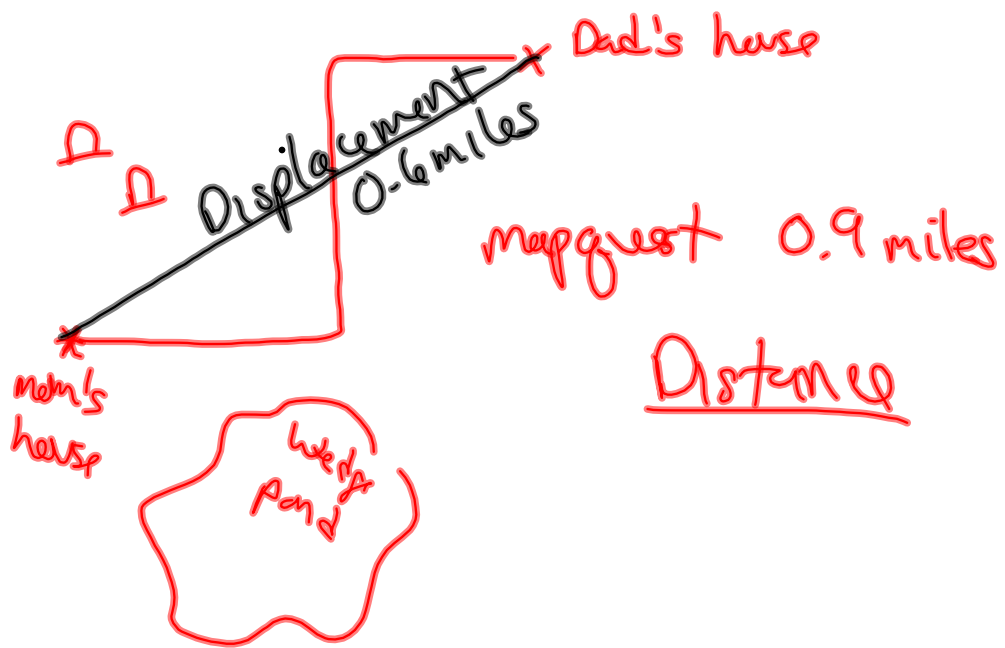


Distance, Displacement, and Location

Distance is how far you have travelled

Displacement is how far you are from where you started
("as the crow flies")

Location is where you are... (x,y) with $(0,0)$ as a starting point



s = position
 x = time

accelerated
motion

$$S(x) = \frac{1}{2}ax^2 + v_0x + s_0$$

constant
acceleration

$$V(x) = ax + v_0$$

$$a(x) = a$$

$$S(x) = \frac{1}{2}ax^2 + v_0x + s_0$$
$$V(x) = ax + v_0$$

an object starts with a velocity of 22m/s and an acceleration of -7m/s^2 . At what time will the object have a velocity of 0m/s?

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$$s_0 = 0\text{m}$$

$$S(x) = \frac{1}{2}ax^2 + v_0x + s_0$$

$$V(x) = ax + v_0$$

$$V(x) = 0 = (-7)x + 22$$

$$7x = 22$$

$$x = \frac{22}{7} \text{ seconds}$$

an object starts with a velocity of 22m/s and an acceleration of -7m/s^2 . How far does the object travel before it comes to rest?

$$s_0 = 0\text{m}$$

$$S(x) = \frac{1}{2}ax^2 + v_0x + s_0$$

$$V(x) = ax + v_0$$

an object starts with a velocity of 22m/s and an acceleration of -7m/s^2 . How far does the object travel before it comes to rest?

$$s_0 = 0\text{m}$$

$$S(x) = \frac{1}{2}ax^2 + v_0x + s_0$$

$$V(x) = ax + v_0$$

$$S(x) = \frac{1}{2}(-7)\left(\frac{22}{7}\right)^2 + 22\left(\frac{22}{7}\right) + 0$$

$$S(x) = \frac{22 \cdot 11}{7} + \frac{22^2}{7}$$

$$S(x) = 103.7 \text{ m}$$

