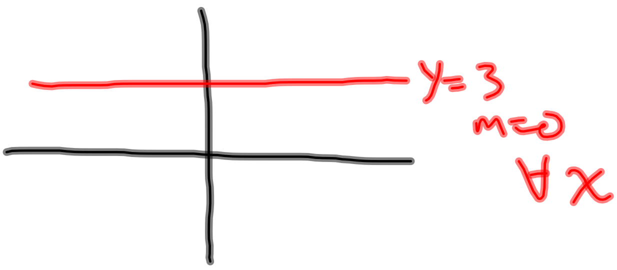
Theorems p189

deriv of a constant
deriv of a power of x
deriv of a sum/difference
deriv of a constant*function

$f(x) = 3$

$f'(x) = 0$



$f(x) = x^n$

$f'(x) = n(x^{n-1})$

$n = \mathbb{R}$

$f(x) = x^2$

$f'(x) = 2x$

$f(x) = g(x) + h(x)$

$f'(x) = g'(x) + h'(x)$

$f(x) = x^2 + 4$

$f'(x) = 2x + 0$

$f(x) = c \cdot g(x)$

$f'(x) = c \cdot g'(x)$

$f(x) = c\, g(x)$

$$f'(x) = \lim_{h \to 0} \frac{c g(x+h) - c g(x)}{h}$$

$$= c \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= c\, g'(x)$$

$f(x) = 3x^2$

$f'(x) = 3(2x)$

$= 6x$

$f(x) = 4x^3 - 2x^2 + 3$

$f'(x) = 12x^2 - 4x + 0$

Product rule

~~Quotient rule~~

$$f(x)=g(x)\cdot h(x)$$
$$f'(x)=g(x)\cdot h'(x)+h(x)\cdot g'(x)$$

The derivative of a product is: "the first times the derivative of the second plus the second times the derivative of the first.

$f(x) = 3x^2$

$f'(x) = 3 \cdot (2x) + x^2(0) = 6x$

$f(x) = x^3 \cdot x^2$

$f'(x) = x^3 \cdot 2x + x^2 \cdot 3x^2 = 2x^4 + 3x^4 = 5x^4$

$f(x) = (3x+2)(2x-1)$

$f'(x) = (3x+2)(2) + (2x-1)(3)$

$= 6x+4 + 6x-3$

$= 12x+1$

$6x^2 - 3x + 4x - 2$

$6x^2 + x - 2$

$12x+1$

~~Product rule~~

Quotient rule

$$f(x) = \frac{g(x)}{h(x)}$$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

the derivative of a quotient is: the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all over the denominator squared.

$$f(x) = \frac{3}{x}$$

$$\frac{-3}{x^2}$$

$$f'(x) = \frac{x \cdot 0 - 3 \cdot 1}{x^2} = \frac{-3}{x^2}$$

$$3x^2$$

$$f(x) = \frac{x^4}{x}$$

$$f'(x) = \frac{x \cdot 4x^3 - x^4 \cdot 1}{x^2} = \frac{4x^4 - x^4}{x^2} = \frac{3x^4}{x^2} = 3x^2$$

$$f(x) = \frac{(x^2-2)}{2x+3}$$

$$f'(x) = \frac{(2x+3)(2x) - (x^2-2)(2)}{(2x+3)^2}$$

$$= \frac{4x^2+6x-(2x^2-4)}{(2x+3)^2} = \frac{4x^2+6x-2x^2+4}{〃}$$

$$= \frac{2x^2+6x+4}{(2x+3)^2} = 2(x^2+3x+2)$$

$$f'(0) = \frac{4}{9}$$

$$= \frac{2(x+2)(x+1)}{(2x+3)^2}$$

Notation: higher order derivatives

$f(x) = 4x^4 - 3x^3 + 2x - 4$ $\qquad \dfrac{dy}{dx}$

$f'(x) = 16x^3 - 9x^2 + 2$ $\qquad \dfrac{d^2y}{dx}$

$f''(x) = 48x^2 - 18x$ $\qquad \dfrac{d^3y}{dx}$

$f'''(x) = 96x - 18$ $\qquad \vdots$

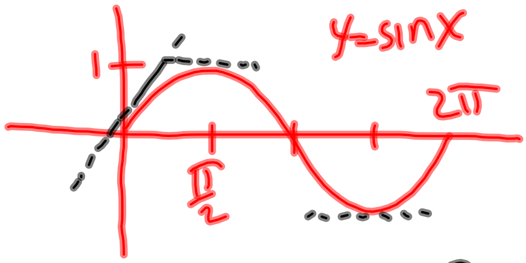
$f^{(4)}(x) = 96$

$f^{(5)}(x) = 0$

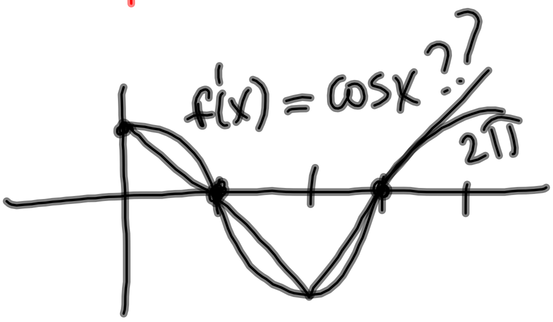
Derivatives of trig functions (pg 200)

$$\frac{d\,[\sin x]}{dx} = \cos x$$

not surprising →



slope of tan to sin x



Derivatives of trig functions (pg 200)

$$\frac{d\,[\sin x]}{dx} = \cos x \qquad \frac{d\,[\sec x]}{dx} = \sec x \tan x$$

$$\frac{d\,[\cos x]}{dx} = -\sin x \qquad \frac{d\,[\csc x]}{dx} = -\csc x \cot x$$

$$\frac{d\,[\tan]}{dx} = \sec^2 x \qquad \frac{d\,[\cot x]}{dx} = -\csc^2 x$$

Exs 1-4

$f(x) = x^2 \tan x$

$f'(x) = x^2 \sec^2 x + \tan x(2x)$

$= x^2 \sec^2 x + 2x \tan x$

$f(x) = \dfrac{\sin x}{1+\cos x}$

$f'(x) = \dfrac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$

$f'(x) = \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} \Rightarrow f'(x) = \dfrac{1+\cos x}{(1+\cos x)^2} = \dfrac{1}{1+\cos x}$

Ex 3

$f(x) = \sec x$ $\qquad y''\left(\frac{\pi}{4}\right)$

$f'(x) = \sec x \tan x$

$f''(x) = \sec x \sec^2 x + \tan x \sec x \tan x$

$= \sec^3 x + \sec x \tan^2(x)$

$f''\left(\frac{\pi}{4}\right) = (\sqrt{2})^3 + \sqrt{2}(1)$

$= 2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$

$\sec\left(\frac{\pi}{4}\right)$

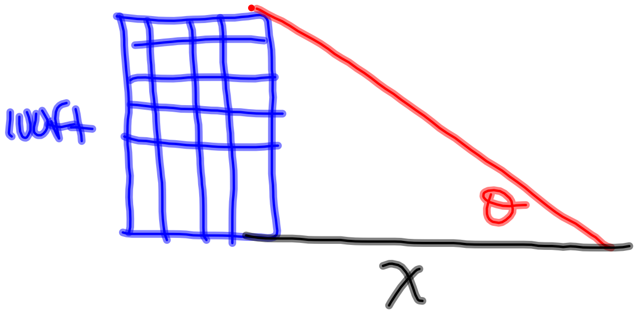
$\frac{1}{\cos\frac{\pi}{4}}$

$\sqrt{2}$

$\tan\left(\frac{\pi}{4}\right) = 1$

Ex 4 pg 202

sun



$\tan\theta = \dfrac{100}{x} \Rightarrow x = \dfrac{100}{\tan x}$

$\cot\theta = \dfrac{x}{100} \Rightarrow x = 100\cot\theta$

$\dfrac{dx}{d\theta} = 100(-\csc^2\theta)$

$= -100\csc^2\theta \Big|_{\pi/4}$

$-100\left(\csc\frac{\pi}{4}\right)^2$

$= -200\ \dfrac{\text{feet}}{\text{rad}}$

Chain Rule! (your friend*) p204

th 3.5.2
ex 1, ex 2
ex 3, ex 4

$f(x) = (\cos x)^3$

$f'(x) = 3(\cos x)^2$??

no.

$\frac{d\,x^n}{dx} = nx^{n-1}$

*K.W. Miller 1976

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$y = f(g(x))$

$g(x) = u$

$y = f(u)$

$y = f(g(x)) = (\cos x)^3$

$f(x) = x^3 = u^3$

$g(x) = \cos x = u$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 3u^2(-\sin x)$$

$$= 3(\cos x)^2(-\sin x) = -3\cos^2 x \sin x$$

Ex 1

$y = 4\cos(x^3)$

$g(x) = x^3 = u$

$f(x) = 4\cos x$

$f(u) = 4\cos u$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$(-4\sin u)\frac{du}{dx}$

$-4\sin u \cdot 3x^2$

$-4(\sin x^3)\,3x^2$

$-12x^2\sin(x^3)$

$$\frac{d\,3x^2}{dx} = 6x \cdot \frac{dx}{dx}$$

$$y = 3(\ )^2$$

$$y' = 6(\ )' \cdot \frac{d(\ )}{dx}$$

$\frac{dw}{dt}$ if $w = \tan x$ $x = 4t^3 + t$

$$\frac{dw}{dx}\frac{dx}{dt} = \sec^2 x \cdot (12t+1)$$

$$\left[\sec^2(4t^3+t)\right](12t+1)$$

$f(t) = \tan(4t^3 + t)$

$f'(t) = [\sec^2(4t^3 + t)](12t + 1)$

$f(x) = (2x+1)^2$

$f'(x) = 2(2x+1)^1 \cdot 2$

$f(x) = x^2$

$f'(x) = 2x \frac{dx}{dx}$

$f(x) = (x^3+1)^2$

$f'(x) = 2(x^3+1)^1 \cdot (3x^2)$

$f(x) = \sin(2x+6)$

$f'(x) = [\cos(2x+6)] \cdot 2$

compositions (not products)

$f(x) = 3x \sin x$

$f'(x) = 3x(-\cos x) + (\sin x)3$

#4 pg 208

$$f(x) = \frac{1}{(x^5 - x + 1)^9}$$

$f'(x) =$

#4 pg 208

$$f(x) = \frac{1}{(x^5 - x + 1)^9} = (x^5 - x + 1)^{-9}$$

$$f'(x) = -9(x^5 - x + 1)^{-10} \cdot (5x^4 - 1)$$

$$= \frac{-9(5x^4 - 1)}{(x^5 - x + 1)^{10}}$$

$y = x^{-9}$

$y' = -9x^{-10}$

$= \frac{-9}{x^{-10}}$

$$f(x) = \left(\sin(3x^2+1)\right)^3$$

$$f'(x) = 3\left(\sin(3x^2+1)\right)^2\left[\cos(3x^2+1)\right](6x)$$

$f(x) = x^3$

$g(x) = \sin x = u$

$h(x) = 3x^2+1 = v$

$$\frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} = \frac{dy}{dx}$$

Homework:

pg 197 5,8,23,25,33,37,41a,42c,45a,61,75

pg 202 5,15,19,26c,27,31

pg 208 7,23,35,41,45,49,65