Homework:

pg 175 1,10,11,21

pg 186 3b,7,9,15,23,27,31,39

1) $y=\frac{1}{2}x^2 \qquad x_0=3 \quad x_1=4$

a) $m_{sec}=\dfrac{\frac{1}{2}(4)^2-\frac{1}{2}(3)^2}{4-3}=\dfrac{8-\frac{9}{2}}{1}=\dfrac{7}{2}$

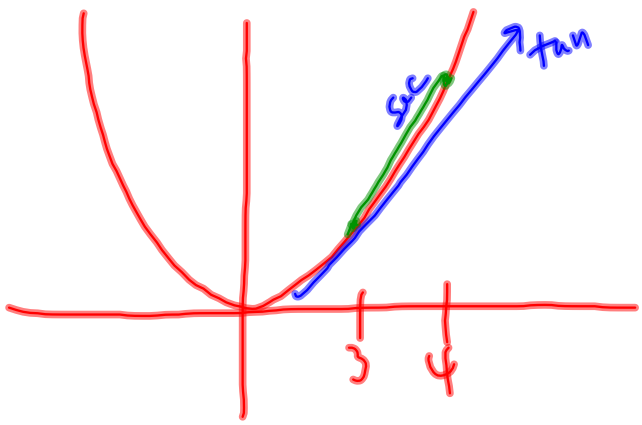
b) $\lim\limits_{x\to 3}\dfrac{f(x)-f(3)}{x-3}=\lim\limits_{x\to 3}\dfrac{\frac{1}{2}x^2-\frac{9}{2}}{x-3}=\lim\limits_{x\to 3}\dfrac{\frac{x^2-9}{2}}{x-3}$

$=\lim\limits_{x\to 3}\dfrac{\frac{(x+3)(x-3)}{2}}{(x-3)}=\lim\limits_{x\to 3}\dfrac{3+x}{2}=3$

1) $y = \frac{1}{2}x^2 \qquad x_0 = 3 \quad x_1 = 4$

c) $\lim_{h\to 0} \frac{\frac{1}{2}(x+h)^2 - \frac{1}{2}x^2}{h} = \lim_{h\to 0} \frac{\frac{1}{2}(x^2+2xh+h^2) - \frac{1}{2}x^2}{h}$

$= \lim_{h\to 0} \frac{xh + \frac{1}{2}h^2}{h} = \lim_{h\to 0} x + \frac{1}{2}h = x$



10) a) at $t=0$ position $= 10$
at $t=3$ position $= 10$ } disp $= 0$

avg vel $= \frac{0}{3} = 0$

b) inst. vel = slope ~~of~~ tan

tan is horiz $(m=0)$ at $t = 0, 2, 4, 8$

c) m tan is max at $t=1$

m tan is min at $t=3$ (largest and (-))

d) $(2.5, 15)$ $(3.5, 5)$ using ruler

$m_{tan} = \frac{5-15}{3.5-2.5} = -10$ = inst vel at $t=3$

11) a) Particle is moving faster at t_0

at t_0 m_{tan} is (+) so vel is (+).

at t_2 tan is horiz (m=0) so vel is 0

b) horiz tan has m=0 so vel = 0 at start

c) from t_0 to t_1 the tangents get steeper (going up) so the slope is increasing, so the vel is increasing.

d) at t_1 the m_{tan} is (+) and at t_2 m_{tan} = 0 so since the slope of the tan is decreasing, the vel is decreasing

21) $s = 6t^4$

$S(2) = 96 \quad S(4) = 1536$

$$\overline{V} = \frac{\Delta S}{\Delta t} = \frac{1536-96}{4-2} = \frac{1440}{2} = 720 \text{ ft/sec}$$

$t_0 = 2 \quad t_1 = t$

$$S'(2) = \lim_{t \to 2} \frac{S(t) - S(2)}{t-2} = \lim_{t \to 2} \frac{6t^4 - 96}{t-2} = 6 \lim_{t \to 2} \frac{t^4 - 16}{t-2}$$

$t^4 - 16 = (t^2+4)(t^2-4)$

$(t^2+4)(t+2)(t-2)$

(over)

$$6\lim_{t\to 2}\frac{(t^2+4)(t+2)\cancel{(t-2)}}{\cancel{(t-2)}} = 6\lim_{t\to 2}(t^2+4)(t+2)$$

$$6(8)(4) = 192 \text{ ft/s}$$

$$f(x) = 6t^4$$

$$f'(x) = 24t^3$$

$$f'(2) = 24(8) = 192$$

3b) $f'(x) = 3x+1 \quad (2,5)$

$(2,5)$ $m_{tan} = 3 = f'(2)$

7) $f(3) = -1 \quad (3,-1)$

$f'(3) = 5 \quad m_{tan} = 5$

$y-(-1) = 5(x-3)$

$y+1 = 5x-15$

$y = 5x-16$ ✓

9) $f(x)=3x^2$

$$f'(x)=\lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h}=\lim_{h\to 0}\frac{3(x^2+2xh+h^2)-3x^2}{h}$$

$$=\lim_{h\to 0}\frac{h\cdot(6x+6h)}{h}=6x$$

$f'(x)=6x$

$f'(3)=m_{tan}=18$

$f(3)=27 \quad (3,27) \quad m=18$

$y-27=18(x-3)$

13) $f(x) = \sqrt{x+1}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}\left(\frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}\right)$$

$$\lim_{h\to 0} \frac{x+h+1-(x+1)}{\left(\sqrt{x+h+1}+\sqrt{x+1}\right)h} = \lim_{h\to 0} \frac{\cancel{h}}{\left(\sqrt{x+h+1}+\sqrt{x+1}\right)\cancel{h}}$$

$$= \frac{1}{\sqrt{x+1}+\sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$

$f(8) = 3 \quad (8,3) \quad m = \frac{1}{4}$

$$f'(8) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$

$y - 3 = \frac{1}{6}(x-8)$

15) $y = \frac{1}{x}$

$$y' = \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{x}{(x+\Delta x)x} - \frac{x+\Delta x}{x(x+\Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{(x+\Delta x)x} \frac{1}{\Delta x} = \lim_{\Delta x \to 0} \frac{-1}{x^2 + x\Delta x} = -\frac{1}{x^2}$$

$$y' = \frac{dy}{dx} = -\frac{1}{x^2}$$

23) a - D
b - F
c - B
d - C
e - A
f - E

27) a) $f(x)=x^2 \quad a=3$

$\lim_{h\to 0} \dfrac{(x+h)^2-x^2}{h}$

$f'(3)=\dfrac{(3+h)^2-9}{h}$

b) $f(x)=\sqrt{x} \quad a=1$

31) $f'(x)=3x^2-2$

$(0,1) \quad m_{tan}=-2$

$y-1=-2(x-0)$

$y=-2x+1$

39) $T(t)$ $T(10) = 115^\circ$

$$\left.\begin{array}{l}(0, 145) \\ (43, 0)\end{array}\right\} \quad m_{tan} = \frac{145-0}{0-43} = \frac{145}{43} = 3.37$$

$$\left.\frac{dT}{dt}\right|_{t=10} = 3.37$$

$$T'(10) = 3.37$$

$$\frac{dT}{dt} = k(T - T_0)$$

$$3.37 = k(115 - 75)$$

$$0.084 = k$$