HW:

Pg 253 9,11,21,27
Pg 260 1,13,21,31,35
Pg 267 1a,2a,5,7d,9a,11,21,29
Pg 275 21,29
pg 284 3,9,19,21,27

9) $x^3 + xy - 2x = 1$

$$3x^2 + x\frac{dy}{dx} + y \cdot 1 - 2 = 0$$

$$x\frac{dy}{dx} = -3x^2 - y + 2$$

$$\frac{dy}{dx} = -3x - \frac{y}{x} + \frac{2}{x}$$

11) $x^2 + y^2 = 100$

$$2x\frac{dx}{dx} + 2y\frac{dy}{dx} = \frac{d(100)}{dx}$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\cancel{2}x}{\cancel{2}y}$$

13) $x^2y + 3xy^3 - x = 3$

$$x^2\frac{dy}{dx} + y\frac{dx^2}{dx} + 3x\frac{dy^3}{dx} + y^3\frac{d\,3x}{dx} - \frac{dx}{dx} = \frac{d\,3}{dx}$$

$$x^2\frac{dy}{dx} + 2xy + 3x\left(3y^2\frac{dy}{dx}\right) + y^3\cdot 3 - 1 = 0$$

$$x^2\frac{dy}{dx} + 9xy^2\frac{dy}{dx} = -2xy - 3y^3 + 1$$

$$\frac{dy}{dx} = \frac{-2xy - 3y^3 + 1}{x^2 + 9xy}$$

#pretend)

$$3x^2 + 2xy - 6y = 0$$

$$6x + 2x\frac{dy}{dx} + y \cdot 2 - 6\frac{dy}{dx} = 0$$

$$(2x-6)\left(\frac{dy}{dx}\right) = -6x - 2y$$

$$\frac{dy}{dx} = -\frac{6x+2y}{2x-6}$$

21) $3x^2-4y^2=7$

$$6x-8y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{6x}{8y}=\frac{3x}{4y}$$

$$\frac{dy}{dx}=\frac{3x}{4y}$$

$$\frac{d^2y}{dx^2}=\frac{3}{4}\left(\frac{y\frac{dx}{dx}-x\frac{dy}{dx}}{y^2}\right)$$

$$=\frac{3}{4}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)$$

$$=\frac{3}{4}\left(\frac{y-x\left(\frac{3x}{4y}\right)}{y^2}\right)$$

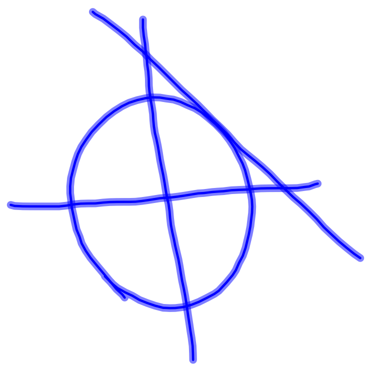
27) $x^2+y^2=1$ $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$2x+2y\frac{dy}{dx}=0$$

$$2y\frac{dy}{dx}=-2x$$

$$\frac{dy}{dx}=-\frac{x}{y} \qquad \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\frac{dy}{dx}=-1 \qquad m_{tan}$$



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31) $y + \ln xy = 1$

$$\frac{dy}{dx} + \frac{1}{xy}\frac{d(xy)}{dx} = 0$$

$$\frac{dy}{dx} + \frac{1}{xy}\left(x\frac{dy}{dx} + y\cdot 1\right) = 0$$

$$\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} + \frac{1}{x} = 0$$

$$\left(1+\frac{1}{y}\right)\frac{dy}{dx} = -\frac{1}{x}$$

$$\frac{y+1}{y}$$

$$\frac{dy}{dx} = -\frac{1}{x}\,\frac{y}{y+1}$$

$$\frac{dy}{dx} = -\frac{y}{x(y+1)}$$

35) $y = x\sqrt[3]{1+x^2}$

$$\ln y = \ln\left(x\sqrt[3]{1+x^2}\right)$$

$$= \ln x + \ln\sqrt[3]{1+x^2}$$

$$\ln y = \ln x + \frac{1}{3}\ln(1+x^2)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{3}\frac{1}{1+x^2}\cdot(2x)$$

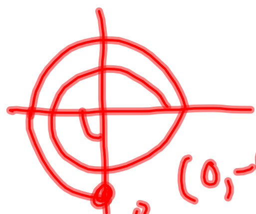
$$\frac{dy}{dx} = \left(\frac{1}{x} + \frac{2x}{3(1+x^2)}\right)y$$

Pg 267

7d) $\sin^{-1}(\sin 630)$

$\sin^{-1}(-1)$

$-\frac{\pi}{2}$



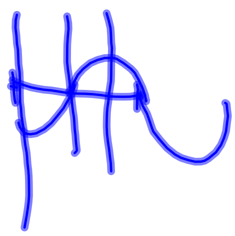
$(0,-1)$

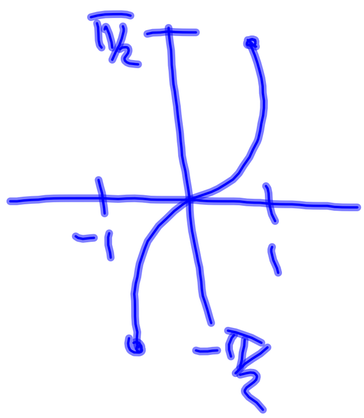
$(\cos, \sin)$

$630°$

$270°$

$90°$





side bar

$360° = 2\pi^R$

$\frac{360°}{2\pi} \quad 1^R$

630 radians

$630 \cdot \frac{360°}{2\pi} = 356256.61° / 360 = 989.601$

$.601 * 360$

$216.3°$

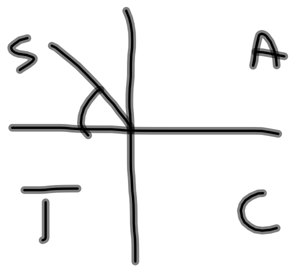
$\sin(216.3) = -.4440$

$\sin^{-1}(-.4440) = -26°$

$$\sin^{-1}(\sin 48^\circ) = 48^\circ$$

$$\sin^{-1}\left(\sin \frac{\pi}{4}\right) = \frac{\pi}{4}$$

$$\sin^{-1}\left(\sin \frac{3\pi}{4}\right) = \frac{\pi}{4}$$



9) $\cos^{-1}(\cos x) = x \quad [0, \pi]$

21) $\sin^{-1}\left(\frac{1}{3}x\right) = y$

$$\frac{dy}{dx} = \left(\frac{1}{\sqrt{1-\left(\frac{1}{3}x\right)^2}}\right)\frac{1}{3}$$

$$1 - \frac{x^2}{9}$$

$$\sqrt{\frac{9-x^2}{9}} = \frac{1}{\frac{\sqrt{9-x^2}}{\cancel{3}}} \cdot \frac{1}{\cancel{3}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{9-x^2}}$$

27) $y = e^x \sec^{-1}(x)$

$$\frac{dy}{dx} = e^x \frac{1}{|x|\sqrt{x^2-1}} + \sec^{-1}x \; e^x$$

$$= e^x\left(\frac{1}{|x|\sqrt{x^2-1}} + \sec^{-1}x\right)$$

29) $x^3 + x\tan^{-1}y = e^y$

$$\frac{de^u}{dx} = e^u\frac{du}{dx}$$

$$3x^2 + x\frac{1}{1+y^2}\frac{dy}{dx} + \tan^{-1}y = e^y\frac{dy}{dx}$$

$$\frac{d\left(\tan^{-1}u\right)}{dx} = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\left(\frac{x}{1+y^2} - e^y\right)\frac{dy}{dx} = -3x^2 - \tan^{-1}y$$

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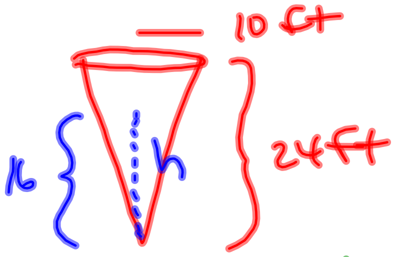
21)



$$\frac{dV}{dt} = 20\,ft^3/min$$

$$\left.\frac{dh}{dt}\right|_{h=16}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{h}{r} = \frac{24}{10} = 2.4$$

$$r = \frac{h}{24}$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2.4}\right)^2 h^3$$

$$V = \frac{\pi}{3\cdot(2.4)^2} h^3$$

$$V = .181 h^3$$

$V = .181h^3$

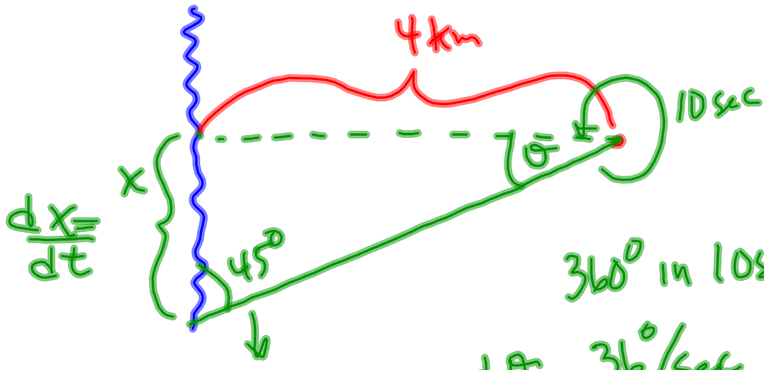
$\frac{dV}{dt} = .181(3h^2)\frac{dh}{dt}$

$20 = .181(3(256))\frac{dh}{dt}$

$\frac{20}{.181(768)} = \frac{dh}{dt}$

$.143 \text{ ft/min} = \frac{dh}{dt}$

29)



$\tan\theta = \frac{x}{4}$

$\theta = \tan^{-1}\left(\frac{x}{4}\right)$

360° in 10sec

$\frac{d\theta}{dt}$ 36°/sec

$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{x}{4}\right)^2}\cdot\frac{1}{4}\frac{dx}{dt}$

@45° ⇒ x = ?

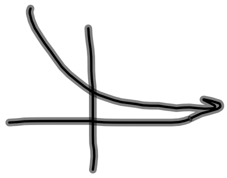
x = 4 duh!

$36° = \frac{1}{1+1}\cdot\frac{1}{4}\frac{dx}{dt}$

$\frac{2\pi}{10} = \frac{1}{8}\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{16\pi}{10} = 5.03$ km/sec pg 284

19) $\lim_{x\to\infty} xe^{-x} = \infty \cdot 0$



$y = xe^{-x}$

$\ln y = \ln(xe^{-x})$

$= \ln x + \ln e^{-x}$

$\ln y = \ln x - x$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} (\ln x - x)$

$\lim_{x\to\infty} \ln x - \lim_{x\to\infty} x$

$\infty - \infty$

$$\lim_{x\to\infty} \frac{\ln x}{e^x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{e^x} = \frac{0}{\infty} = 0$$

21) $\lim_{x \to \infty} x \sin\frac{\pi}{x} = \infty \cdot 0$

$\lim_{x \to \infty} \frac{x}{\csc\frac{\pi}{x}} = \frac{\infty}{\infty}$

$\lim_{x \to \infty} \frac{1}{\csc\frac{\pi}{x}\cot\frac{\pi}{x}} = \frac{1}{(\infty)(-\infty)}$

$= 0$

this is still not correct

$$\lim_{x\to\infty} x \sin\frac{\pi}{x} = \infty \cdot 0$$

let $u = \frac{\pi}{x}$ $\therefore x = \frac{\pi}{u}$ as $x \to \infty$ $u \to 0$

$$\lim_{u\to 0} \frac{\pi}{u} \sin u$$ (use small ∠ approx!!)

$$\lim_{u\to 0} \frac{\pi}{u} \cdot u = \lim_{u\to 0} \pi = \pi$$

27) $\lim_{x\to 0}(e^x+x)^{\frac{1}{x}} = 1^\infty \neq 1$ This is a tendency towards 1, not $=1$

$y = (e^x+x)^{\frac{1}{x}}$

$\ln y = \frac{1}{x}\ln(e^x+x)$

$$\lim_{x\to 0}\ln y = \lim_{x\to 0}\frac{1}{x}\ln(e^x+x) = \lim_{x\to 0}\frac{\ln(e^x+x)}{x} = \frac{\ln 1}{0} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\frac{1}{e^x+x}(e^x+1)}{1} = \lim_{x\to 0}\frac{e^x+1}{e^x+x} = \frac{1+1}{1+0} = 2$$

So as $x\to 0$ $\ln y \to 2$ $e^{\ln y} \to e^2$

$y \to e^2$ ☺