HW: page 521 5,9,15,19,27,31

5) $\int x \sin 2x\,dx$ let $u = x$ $du = dx$

$dv = \sin 2x\,dx$ $v = -\frac{1}{2}\cos 2x$

$$\int u\,dv = -\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x\,dx$$

$$-\frac{1}{2}x\cos 2x + \frac{1}{2}\cdot\frac{1}{2}\sin 2x + C$$

$$-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C$$

9) $\int \sqrt{x} \ln x \, dx$ let $u = \ln x$ $du = \frac{1}{x} dx$
$dv = \sqrt{x} dx$ $v = \frac{2x^{3/2}}{3}$

$$\int u\,dv = \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{3/2}\frac{1}{x}\,dx$$

$$-\frac{2}{3}\int x^{\frac{1}{2}}\,dx$$

$$\frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\frac{x^{3/2}}{3/2} + C$$

$$\frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$$

$$\frac{2}{9}x^{3/2}\left(3\ln x - 2 + C\right)$$

15) $\int \sin^{-1}x\,dx$ $\quad u=\sin^{-1}x$ $\quad du=\frac{1}{\sqrt{1-x^2}}dx$ $\quad dv=dx$ $\quad v=x$

$\int u\,dv = uv - \int v\,du$

$\int u\,dv = x\sin^{-1}x - \int \frac{x\,dx}{\sqrt{1-x^2}}$

$+\frac{1}{2}\int (1-x^2)^{-\frac{1}{2}}(-2x\,dx)$ $\quad \int u^n\,du$

$+\frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + C$

$x\sin^{-1}x+\sqrt{1-x^2}+C$

19) $\int e^x \sin x\,dx$ $\quad u=\sin x \quad dv=e^x dx$ $\quad du=\cos x\,dx \quad v=e^x$

$$\int u\,dv = e^x \sin x - \int e^x \cos x\,dx$$

$\int e^x \cos x\,dx$ $\quad u=\cos x \quad dv=e^x dx$ $\quad du=-\sin x\,dx \quad v=e^x$

$$\int u\,dv = e^x \cos x + \int e^x \sin x\,dx$$

$$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx + \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x + C$$

27) $\int x^3 e^{x^2} dx$ let $u = x^2$ $du = 2x\,dx$

$dv = e^{x^2} x\,dx$ $v = \frac{1}{2}e^{x^2}$

$$\int u\,dv = \frac{1}{2}x^2 e^{x^2} - \frac{1}{2}\int e^{x^2} 2x\,dx$$

$$= \frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2} + C$$

$y = \frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2} + C$

$y' = \frac{1}{2}(x^2 e^{x^2} 2x + e^{x^2} 2x) - \frac{1}{2}e^{x^2} 2x$

$= x^3 e^{x^2} + x e^{x^2} - x e^{x^2}$

$= x^3 e^{x^2}$

31) $\int_1^e x^2 \ln x \, dx$ let $u = \ln x$ $du = \frac{1}{x} dx$

$dv = x^2 dx$ $v = \frac{x^3}{3}$

$$\int u\,dv = \frac{x^3 \ln x}{3} - \frac{1}{3}\int x^3 \frac{1}{x} dx$$

$$= \frac{x^3 \ln x}{3} - \frac{1}{3}\left(\frac{x^3}{3} + C\right)$$

$$= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \Big|_1^e$$

$$= \frac{1}{3}e^3 - \frac{1}{9}e^3 - \left(-\frac{1}{9}\right) = \frac{2}{9}e^3 + \frac{1}{9}$$