

HW: page 542: 9,15,21

$$9) \int \frac{dx}{x^2+3x-4} = \frac{1}{5} \left[ \int \frac{dx}{x-1} - \int \frac{dx}{x+4} \right]$$

$$\left[ \frac{1}{(x-1)(x+4)} = \frac{A}{x-1} + \frac{B}{x+4} \right]^{(x+4)(x-1)} = \frac{1}{5} (\ln|x-1| - \ln|x+4|)$$

$$1 = A(x+4) + B(x-1)$$

$$0 = A + B$$

$$1 = 4A - B$$

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$$1 = 5A$$

$$A = \frac{1}{5} \quad B = -\frac{1}{5}$$

$$= \frac{1}{5} \ln \left| \frac{x-1}{x+4} \right| + C$$

$$15) \int \frac{x^2+2}{x+2} dx =$$

$$= \int \left( x - 2 + \frac{6}{x+2} \right) dx$$

$$= \frac{x^2}{2} - 2x + 6 \ln|x+2| + C$$

$$\begin{array}{r} x-2 \\ x+2 \overline{) x^2 + 0x + 2} \\ \underline{\ominus x^2 + 2x} \phantom{+ 2} \\ -2x + 2 \\ \underline{\ominus -2x - 4} \\ 6 \end{array}$$

$$21) \int \frac{2x^2+3}{x(x-1)^2} dx$$

$$\left[ \frac{2x^2+3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \right] (x)(x-1)^2$$

$$\begin{aligned} 2x^2+3 &= A(x-1)^2 + Bx(x-1) + Cx \\ &= A(x^2-2x+1) + Bx^2-Bx + Cx \\ &= Ax^2 - 2Ax + A + Bx^2 - Bx + Cx \end{aligned}$$

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$$2 = A+B$$

$$0 = -2A - B + C$$

$$3 = A$$

$$\begin{aligned} 2 &= A+B \\ 0 &= -2A-B+C \\ 3 &= A \end{aligned}$$

$$2 = 3 + B$$

$$B = -1$$

$$0 = -2(3) - (-1) + C$$

$$0 = -6 + 1 + C$$

$$C = 5$$

$$\int \frac{2x^2+3}{x(x-1)^2} dx$$

$$= \int \frac{3}{x} - \frac{1}{x-1} + \frac{5}{(x-1)^2} dx$$

$$= 3 \ln|x| - \ln|x-1| + 5 \frac{(x-1)^{-1}}{-1} + C$$

$$= 3 \ln|x| - \ln|x-1| - \frac{5}{x-1} + C$$